EXAMPLE:

An investigator is told that a closed vessel holds a pure rare (noble) gas. She measures the density of the gas in the vessel at 298 K and 1 atm pressure and finds it to be 0.8252 g L^{-1}. (a) What gas does the investigator think is in the vessel? (b) After turning in her report, she is told that the vessel actually contains a mixture of He and Ar. What percentage of the gas in the container is He?

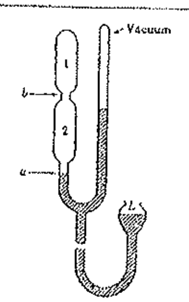
EXAMPLE:

A container of fixed volume holds two ideal gases, denoted as A and B, such that the mole fraction of gas A is X_A=1/3. At a given temperature, the pressure in the container, p_1 is measured. Two additional moles of one of the gases are now added to the container at the same temperature. The new pressure, p_2 is measured. It is found that the ratio p_2/p_1 = 11/9. How many moles of A and B were originally present in the container?

EXAMPLE:

He is contained at 30.2°C in the system shown below. The leveling bulb L can be raised so as to fill the lower bulb with Hg and force the gas into the upper part of the device. The volume of bulb 1 to the mark *b* is 100.5 cm^3 and the volume of bulb 2 between the marks *a* and *b* is 110.0 cm^3. The pressure exerted by the He is measured by the difference between the Hg levels in the device and in the evacuated arm of the manometer. When the Hg level is at *a*, the pressure is 20.14 mm Hg.

What is the mass of He in the system?



EXAMPLE:

He is contained at 30.2°C in the same type of system as the previous problem, except that this time the volume of bulb 1 (V_1) is not known. The volume of bulb 2 (V_2) between the marks *a* and *b* is 110.0 cm^3. When the Hg level is at *a*, the pressure is 15.42 mm Hg. When the Hg level is raised to *b*, the gas pressure rises to 27.35 mm Hg.

(a) What is the mass of He in the system?

(b) What is the volume of bulb 1?

EXAMPLE:

An investigator is told that a closed vessel holds a pure rare (noble) gas. She measures the density of the gas in the vessel at 298 K and 1 atm pressure and finds it to be 0.8252 g L^{-1}. (a) What gas does the investigator think is in the vessel? (b) After turning in her report, she is told that the vessel actually contains a mixture of He and Ar. What percentage of the gas in the container is He?

Principles and equations for this problem:
Assume ideal gas equation of state:
$pV = nRT$ or $V/n = RT/p$
density = mass/volume

Solution:

$RT/p = 0.08205 \text{ L atm mol}^{-1}\text{K}^{-1} \times 298 \text{ K}/1 \text{ atm}$
$= 24.4509 \text{ L mol}^{-1} = V/n$

$n/V = (\text{mass}/\text{molar mass})/\text{volume}$
$= \text{density}/\text{molar mass} = 0.8252 \text{ g L}^{-1}/M$
$M = (V/n) \text{ L mol}^{-1} \times 0.8252 \text{ g L}^{-1}$
$M = 24.4509 \text{ L mol}^{-1} \times 0.8252 \text{ g L}^{-1} = 20.18 \text{g mol}^{-1}$
(a)This is the molar mass of Ne, so she thinks.

(b) It is actually a mixture of He and Ar; what is %He? Use definition of the word "average":

$\langle M \rangle = 20.18 \text{g mol}^{-1} = f_{He}M_{He} + f_{Ar}M_{Ar}$
$= f_{He}M_{He} + (1-f_{He})M_{Ar}$
$= f_{He}4.003 + (1-f_{He})39.948$

$f_{He} = 0.5500$
55% He ***Answer***

EXAMPLE:

A container of fixed volume holds two ideal gases, denoted as A and B, such that the mole fraction of gas A is $X_A=1/3$. At a given temperature, the pressure in the container, p_1 is measured. Two additional moles of one of the gases are now added to the container at the same temperature. The new pressure, p_2 is measured. It is found that the ratio $p_2/p_1 = 11/9$. How many moles of A and B were originally present in the container?

Principles and equations for this problem:
Ideal gas equation of state: $pV = nRT$

Solution:

$$p_1 = \frac{(n_A+n_B)RT}{V}$$

After adding 2 moles of either gas A or B,

$$p_2 = \frac{(n_A+n_B+2)RT}{V}$$

T and V unchanged, divide one eqn by the other:

$$\frac{p_2}{p_1} = \frac{(n_A+n_B+2)}{(n_A+n_B)} = \frac{11}{9} \qquad (*)$$

Also given,

$X_A = n_A/(n_A+n_B) = 1/3$

Rearrange this to $n_A + n_B = 3n_A$

Substitute this into the eqn (*) to get

$$\frac{(3n_A+2)}{(3n_A)} = \frac{11}{9}$$

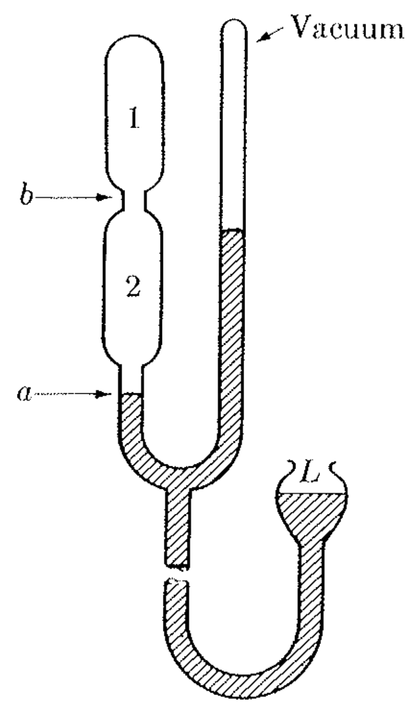
$n_A = 3$ ***Answer***

$n_B = 6$

EXAMPLE:

He is contained at 30.2°C in the system shown below. The leveling bulb L can be raised so as to fill the lower bulb with Hg and force the gas into the upper part of the device. The volume of bulb 1 to the mark b is 100.5 cm^3 and the volume of bulb 2 between the marks a and b is 110.0 cm^3. The pressure exerted by the He is measured by the difference between the Hg levels in the device and in the evacuated arm of the manometer. When the Hg level is at a, the pressure is 20.14 mm Hg.

What is the mass of He in the system?



Principles and equations for this problem:
Assume ideal gas equation of state:
$pV = nRT$

Solution:

Total volume = 100.5 +110.0 =210.5 cm^3.

$$n = \frac{pV}{RT} = \frac{(20.14/760)\ \text{atm}\times(0.2105)\ \text{L}}{.08205\ \text{L atm mol}^{-1}\text{K}^{-1}\times(30.2+273.15)\text{K}}$$

$n = 2.24\times10^{-4}$ mol

mass of He = 2.24×10^{-4} mol$\times$4.0026 g mol^{-1}

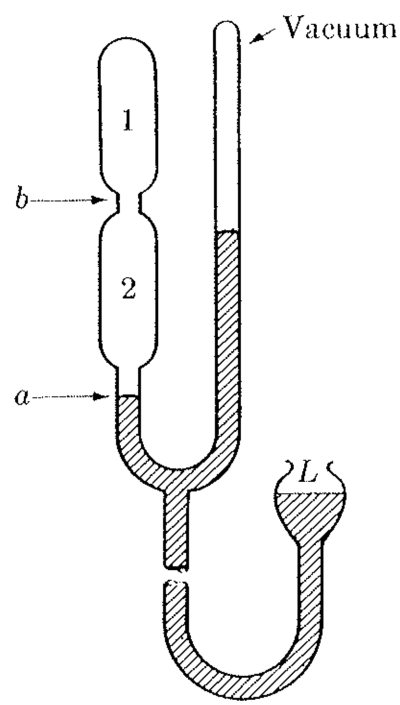
Answer

EXAMPLE:

He is contained at 30.2°C in the same type of system as the previous problem, except that this time the volume of bulb 1 (V_1) is not known. The volume of bulb 2 (V_2) between the marks a and b is 110.0 cm^3. When the Hg level is at a, the pressure is 15.42 mm Hg. When the Hg level is raised to b, the gas pressure rises to 27.35 mm Hg.

(a) What is the mass of He in the system?

(b) What is the volume of bulb 1?



Principles and equations for this problem:
Assume ideal gas: $pV = nRT$
Solution:
The amount of He in the system (*n* mol) is a constant since no He escaped during the raising of the Hg level.
Condition 1: When the Hg level is at *a*, the pressure is 15.42 mm Hg.

$$n = \frac{pV}{RT} = \frac{(15.42/760)\text{ atm}\times(V_1+0.1100)\text{ L}}{.08205\text{ L atm mol}^{-1}\text{K}^{-1}\times(30.2+273.15)\text{K}}$$

Condition 2: When the Hg level is at *b*, the pressure is 27.35 mm Hg.

$$n = \frac{pV}{RT} = \frac{(27.35/760)\text{ atm}\times V_1\text{ L}}{.08205\text{ L atm mol}^{-1}\text{K}^{-1}\times(30.2+273.15)\text{K}}$$

Divide one equation by the other to find V_1:

$$1 = \frac{(15.42/760)\text{ atm}\times(V_1+0.1100)\text{ L}}{(27.35/760)\text{ atm}\times V_1\text{ L}}$$

$V_1 = 0.1422$ L ***Answer***
Into eqn. 2 goes V_1 to get n=2.055×10^{-4} mol
mass of He = 2.055×10^{-4} mol$\times$4.0026 g mol^{-1}.
Answer

EXAMPLE:
A container is partitioned into two compartments of equal volume V. The upper compartment contains H_2 under a pressure of 1 atm; the lower compartment is evacuated. One arm of a manometer is covered by a thin Pd foil and is connected to the H_2-filled compartment. The other arm of the manometer is open to a pressure of 1 atm, which is kept constant during the experiment, as is the temperature. At the beginning of the experiment, the mercury levels in the two arms of the manometer stand at the same height. This is possible because the Pd membrane is permeable to H_2 but not to other gases, and so the membrane does not block the entrance of H_2 to the manometer arm. (You may neglect the volume of the manometer arm.) After equilibrium is reached, what will be the final positions of the mercury levels in the manometer?

EXAMPLE:
A different experiment is performed. In this experiment, the lower compartment contains N_2 (which can not pass the Pd foil) under 1 atm pressure. At the beginning of the experiment, the mercury levels stand at the same height. The partition is removed and the gases mix throughout the container. After equilibrium is reached, what will be the final positions of the mercury levels in the manometer?

EXAMPLE:
Assuming that air has a mean molar mass of 28.8 g mol^{-1}, and that the atmosphere is isothermal at 25°C, compute the barometric pressure at Mile High Stadium in Denver, CO which is 1 mile above sea level. The pressure at sea level may be taken to be 760 mm Hg.

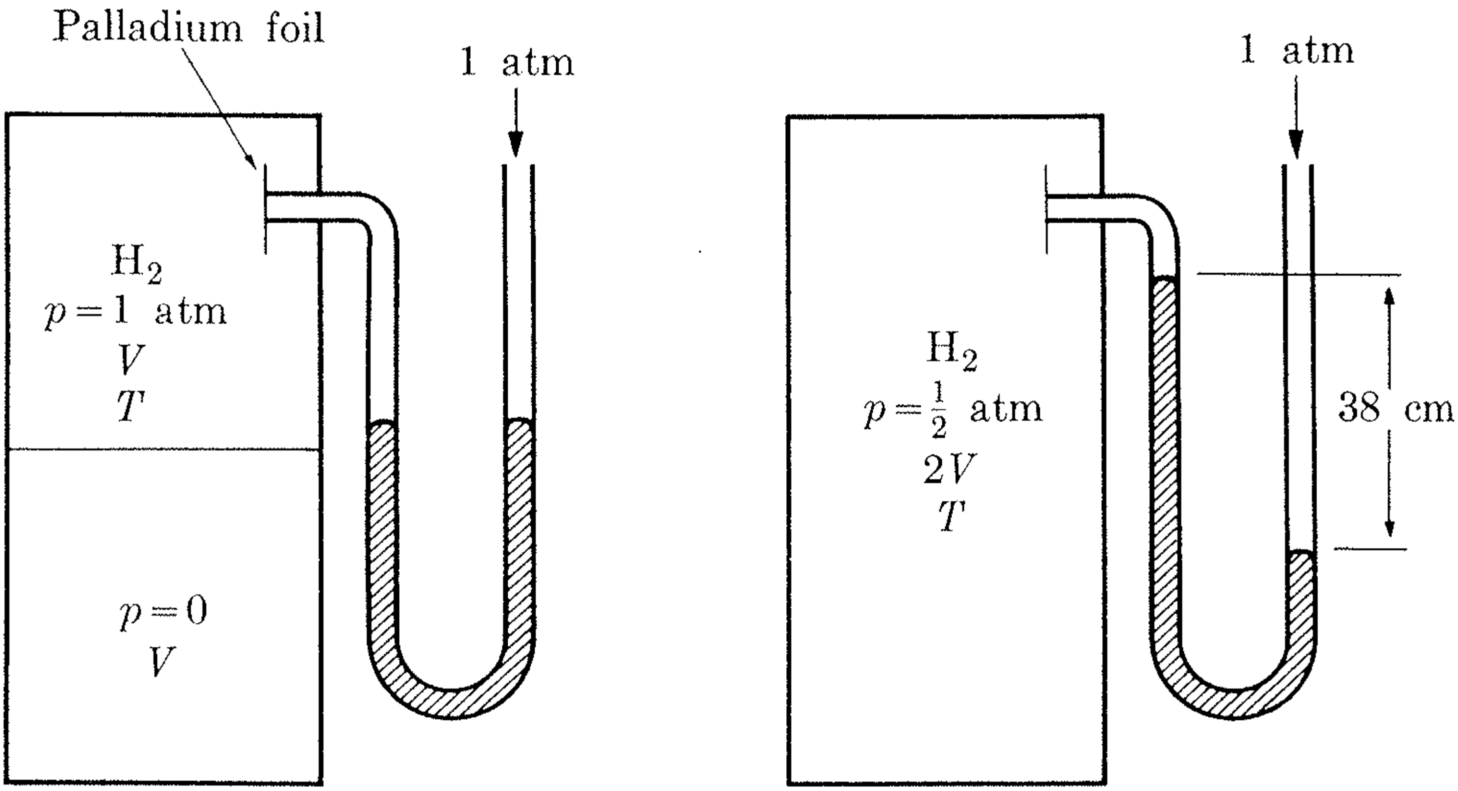
Compute the barometric pressure at the top of Mt. Evans, 14,260 ft above sea level.

Some conversion factors and constants:
1 mile = 160934.4 cm
1 ft = 12 in × 2.54 cm/in = 30.48 cm
$g = 980.665\ cm\ s^{-2}$
$R = 8.31451\ J\ K^{-1}\ mol^{-1}$
$J = kg\ m^2\ s^{-2}$

EXAMPLE:

A container is partitioned into two compartments of equal volume V. The upper compartment contains H_2 under a pressure of 1 atm; the lower compartment is evacuated. One arm of a manometer is covered by a thin Pd foil and is connected to the H_2-filled compartment. The other arm of the manometer is open to a pressure of 1 atm, which is kept constant during the experiment, as is the temperature. At the beginning of the experiment, the mercury levels in the two arms of the manometer stand at the same height. This is possible because the Pd membrane is permeable to H_2 but not to other gases, and so the membrane does not block the entrance of H_2 to the manometer arm. (You may neglect the volume of the manometer arm.) After equilibrium is reached, what will be the final positions of the mercury levels in the manometer?

Draw a picture!



$p_{initial} = n_{H2}(RT/V) = 1$ atm
$p_{final} = n_{H2}(RT/2V) = ½$ atm$=$ 38 cm Hg.

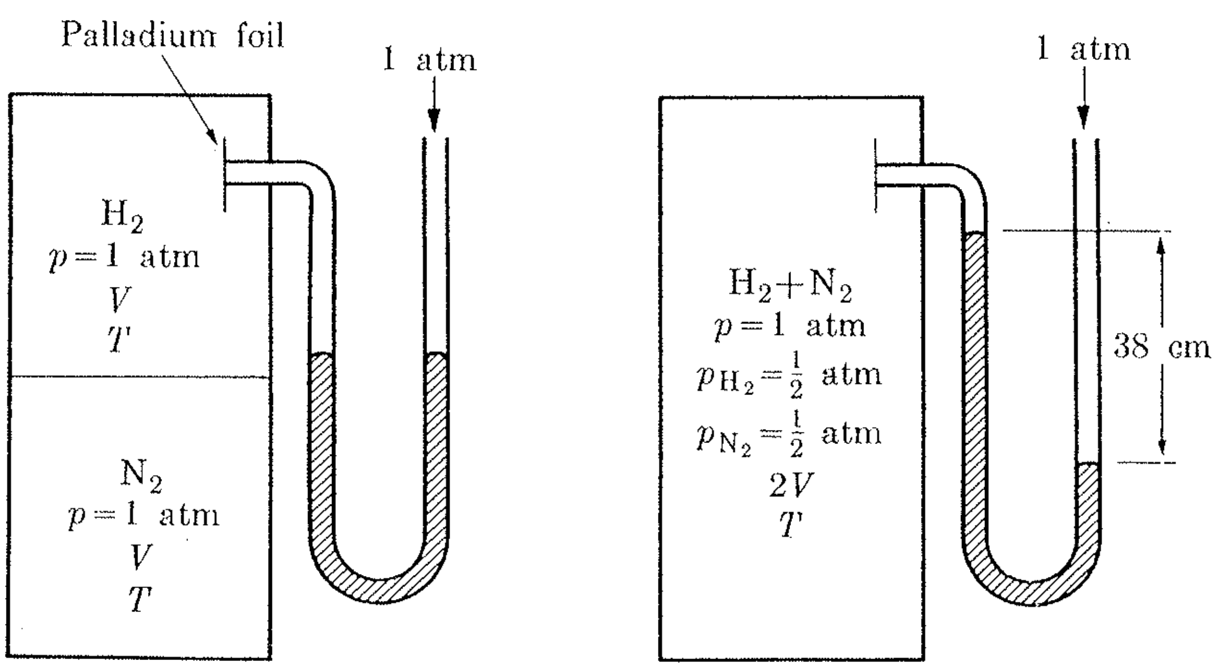
Answer

Since the volume available to H_2 gas is now 2V, the pressure in the container has fallen to ½ of its original value

[Note: The manometer read total pressure before and after the partition was removed.]

EXAMPLE:

A different experiment is performed. In this experiment, the lower compartment contains N_2 (which can not pass the Pd foil) under 1 atm pressure. At the beginning of the experiment, the mercury levels stand at the same height. The partition is removed and the gases mix throughout the container. After equilibrium is reached, what will be the final positions of the mercury levels in the manometer?



$p_{initial} = n_{H2}(RT/V) = 1$ atm

$p_{H2\ (final)} = n_{H2}(RT/2V) = ½$ atm ***Answer***

$p_{N2\ (final)} = n_{N2}(RT/2V) = ½$ atm

$p_{final} = p_{H2} + p_{H2} = 1$ atm

Note that the total pressure in the container does not change upon removal of the partition. The mercury levels in the manometer in this experiment are exactly the same as in the first example where the lower compartment was evacuated.

The H_2 behaves exactly as if the N_2 were not present. The manometer read total pressure before the partition was removed, and *partial pressure* of H_2 in the mixture afterwards.

Later on, we will see that it is the *partial pressures* of the gases in the mixture which are significant in physical and chemical equilibria.

EXAMPLE:

Assuming that air has a mean molar mass of 28.8 g mol^{-1}, and that the atmosphere is isothermal at 25°C, compute the barometric pressure at Mile High Stadium in Denver, CO which is 1 mile above sea level. The pressure at sea level may be taken to be 760 mm Hg.

Compute the barometric pressure at the top of Mt. Evans, 14,260 ft above sea level.

Some conversion factors and constants:

1 mile = 160934.4 cm

1 ft = 12 in $\times$ 2.54 cm/in = 30.48 cm

$g = 980.665\ cm\ s^{-2}$

$R = 8.31451\ J\ K^{-1}\ mol^{-1}$

$J = kg\ m^2\ s^{-2}$

Principles and equations for this problem:
Assume ideal gas equation of state:
$pV = nRT$
We derived the barometric distribution eqn
$p/p_0 = \exp[- Mgz/RT]$ for an ideal gas in a gravitational field at constant temperature T.
Incidentally, the integration could have been carried out between any two limits, leading to
$p_2/p_1 = \exp[- Mg(z_2-z_1)/RT]$ for an ideal gas in a gravitational field at constant temperature T.

Solution:
$p = p_0\exp[- Mgz/RT]$
$p \text{ mm Hg} = 760 \text{ mm Hg} \times \exp[- Mgz/RT]$
where the mean molar mass $M = 28.8 \text{ g mol}^{-1}$.

At Mile-High Stadium:
$Mgz/RT =$

$$\frac{0.028.8 \text{ kg mol}^{-1}\times 9.80665 \text{ m s}^{-2}\times 1609.344 \text{ m}}{8.31451 \text{ kg m}^2\text{s}^{-2}\text{ K}^{-1}\text{ mol}^{-1}\ (25+273.15)\text{K}}$$

$= 0.183$
$p = 760 \times e^{-0.183} = 633 \text{ mm Hg}$ ***Answer***

At top of Mt. Evans: 14260x0.3048 = 4346.4 m

Mgz/RT =

$$\frac{0.028.8 \text{ kg mol}^{-1} \times 9.80665 \text{ m s}^{-2} \times 4346.4 \text{ m}}{8.31451 \text{ kg m}^2\text{s}^{-2} \text{ K}^{-1} \text{ mol}^{-1} (25+273.15)\text{K}}$$

= 0.495

$p = 760 \times e^{-0.495} = 463$ mm Hg ***Answer***

Chemistry 342

Problem Set 1

1. Five grams of ethane are confined in a bulb of one liter capacity. The bulb is so weak that it will burst if the pressure exceeds 10 atm. At what temperature will the pressure of the gas reach the bursting value? Answer in °Celsius.

2. The coefficient of thermal expansion α is defined by $\alpha = (1/V)(\partial V/\partial T)_p$. Using the equation of state, compute the value of α for an ideal gas. The coefficient of compressibility β is defined by $\beta = -(1/V)(\partial V/\partial p)_T$. Compute the value of β for an ideal gas. For an ideal gas, express the derivative $(\partial p/\partial T)_V$ in terms of α and β.

3. A mixture of nitrogen and water vapor is admitted to a flask which contains a solid drying agent. Immediately after admission, the pressure in the flask is 760 mm. After standing some hours, the pressure reaches a steady value of 745 mm. (a) Calculate the composition, in mole percent, of the original mixture. (b) If the experiment is done at 20°C and the drying agent increase in weight by 0.150 g what is the volume of the flask? (The volume occupied by the drying agent may be ignored.)

4. A mixture of oxygen and hydrogen is analyzed by passing it over hot copper oxide and through a drying tube. Hydrogen reduces the CuO to metallic Cu. Oxygen then reoxidizes the copper back to CuO. 100 cm^3 of the mixture measured at 25°C and 750 mm yields 84.5 cm^3 of dry oxygen measured at 25°C and 750 mm after passage over CuO and the drying agent. What is the original composition of the mixture? {Hint: First write balanced chemical equations for the reactions.}

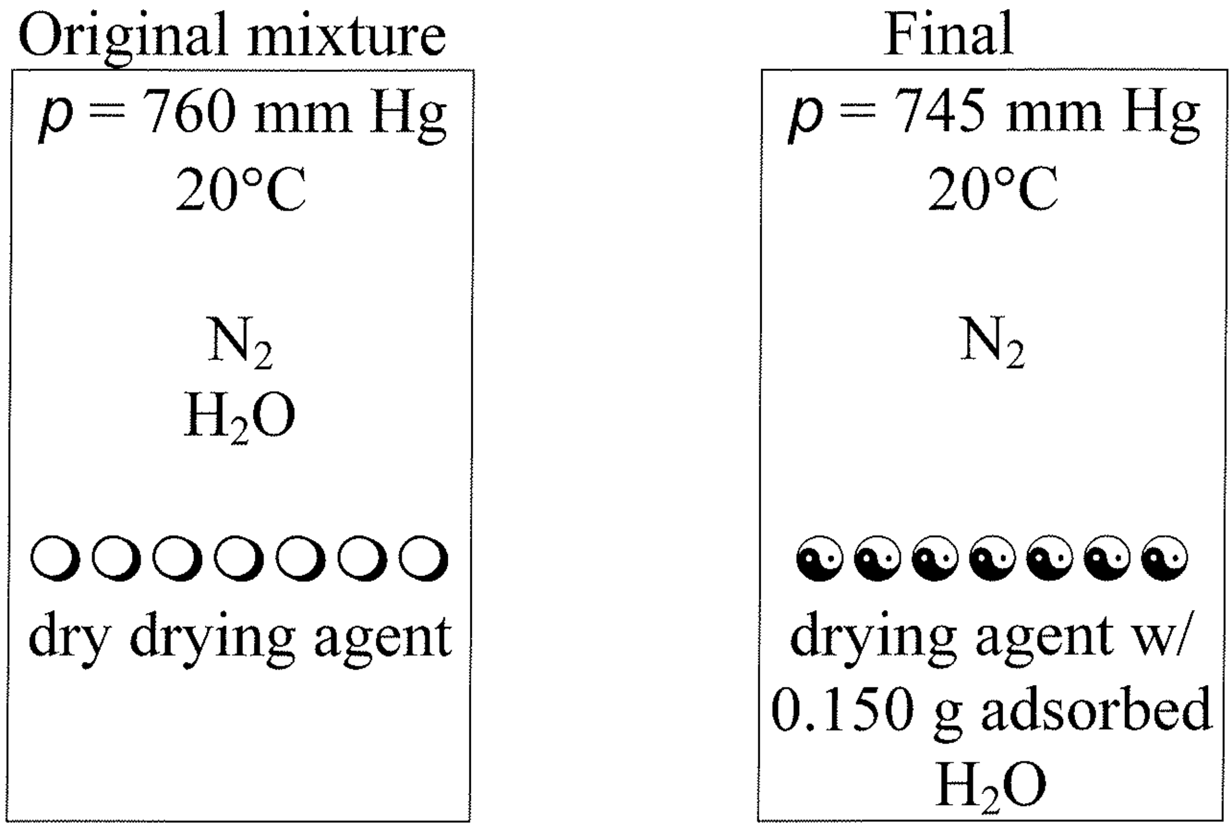
5. A balloon having a capacity of 10,000 m^3 is filled with helium at 20°C and 1 atm pressure. If the balloon is loaded with 80% of the load that it can lift at ground level, at what height will the balloon come to rest? Assume that the volume of the balloon is constant, the atmosphere isothermal, 20°C; the molecular weight of air is 28.8 and the ground level pressure is 1 atm. The mass of the balloon is 1.3×10^6 g.

6. For a gas mixture in a gravity field, it can be shown that each of the gases obeys the distribution law independent of the others. For each gas, $p_i = p_{i0}\exp[-M_i gz/RT]$ where p_i is the partial pressure of the *i*th gas in the mixture at the height z, p_{i0} is the partial pressure of the gas at ground level, and M_i is the molecular weight of the gas. The approximate composition of the atmosphere at sea level is given in the table below:

EXAMPLE

Problem 3. A mixture of nitrogen and water vapor is admitted to a flask which contains a solid drying agent. Immediately after admission, the pressure in the flask is 760 mm. After standing some hours, the pressure reaches a steady value of 745 mm. (a) Calculate the composition, in mole percent, of the original mixture. (b) If the experiment is done at 20°C and the drying agent increase in weight by 0.150 g what is the volume of the flask? (The volume occupied by the drying agent may be ignored.)

Draw a picture



Question: (a) mole percent N_2= ? (b) V = ?

Principles and Definitions:

Definition of partial pressure: $p_{N2} = x_{N2}p$

Dalton's law of partial pressures:

$p_{N2} + p_{H2O} = p$ $p_{N2} = p$

Assume the gases behave ideally: $pV = nRT$

For an ideal gas the partial pressure of a gas in a mixture of gases is the pressure that would be exerted by the gas if it had been alone by itself in the same volume and temperature.

Solution:

In the same V and same $T = (20+273.15)$ K,

$p_{N2} = p = 745$ mm Hg.

Thus, $p_{N2} = 745$ mm Hg in the original mixture.

(and $p_{H2O} = 760-745 = 15$ mm Hg).

From the definition of partial pressure:

$\{ p_{N2} = x_{N2}p \}$ applies in the original mixture,

where $p_{N2} = 745$ mm Hg and $p = 760$ mm Hg,

from which equation we find

$x_{N2} = p_{N2}/p = 745/760 = 0.980$

(a): $100\times x_{N2} = 98\%$ **Answer**

(b) Mass of water vapor in original mixture = increase in weight of the drying agent = 0.150 g H_2O. Assume water vapor behaves ideally in the original mixture:

$p_{H2O} =$ 15 mm Hg, which is the pressure 0.150 g H_2O would exert if by itself in the volume V and temperature $T = 293.15$ K. Substitute these data into $V = nRT/p$ to obtain

$V = (0.150 \text{ g}/ 18.0 \text{ g mol}^{-1})$

$\times 8.20578\times 10^{-2}$ L atm K^{-1} mol$^{-1}\times 293.15$ K

$\div(15 \text{ mm Hg}\times 1 \text{ atm}/760 \text{ mm Hg}) = 10.2$ L

Answer

Not asked for, but we can also find the amount of N_2:

(1) We can use the ideal gas law in the final picture:

$$n = pV/RT$$

$= 745$ mm Hg × (1 atm/760 mm Hg) × 10.2 L ÷ {8.20578×10^{-2} L atm K^{-1} mol^{-1} × 293.15 K}

$= 0.414$ mol N_2

or (2) we can use the definition of mole fraction:

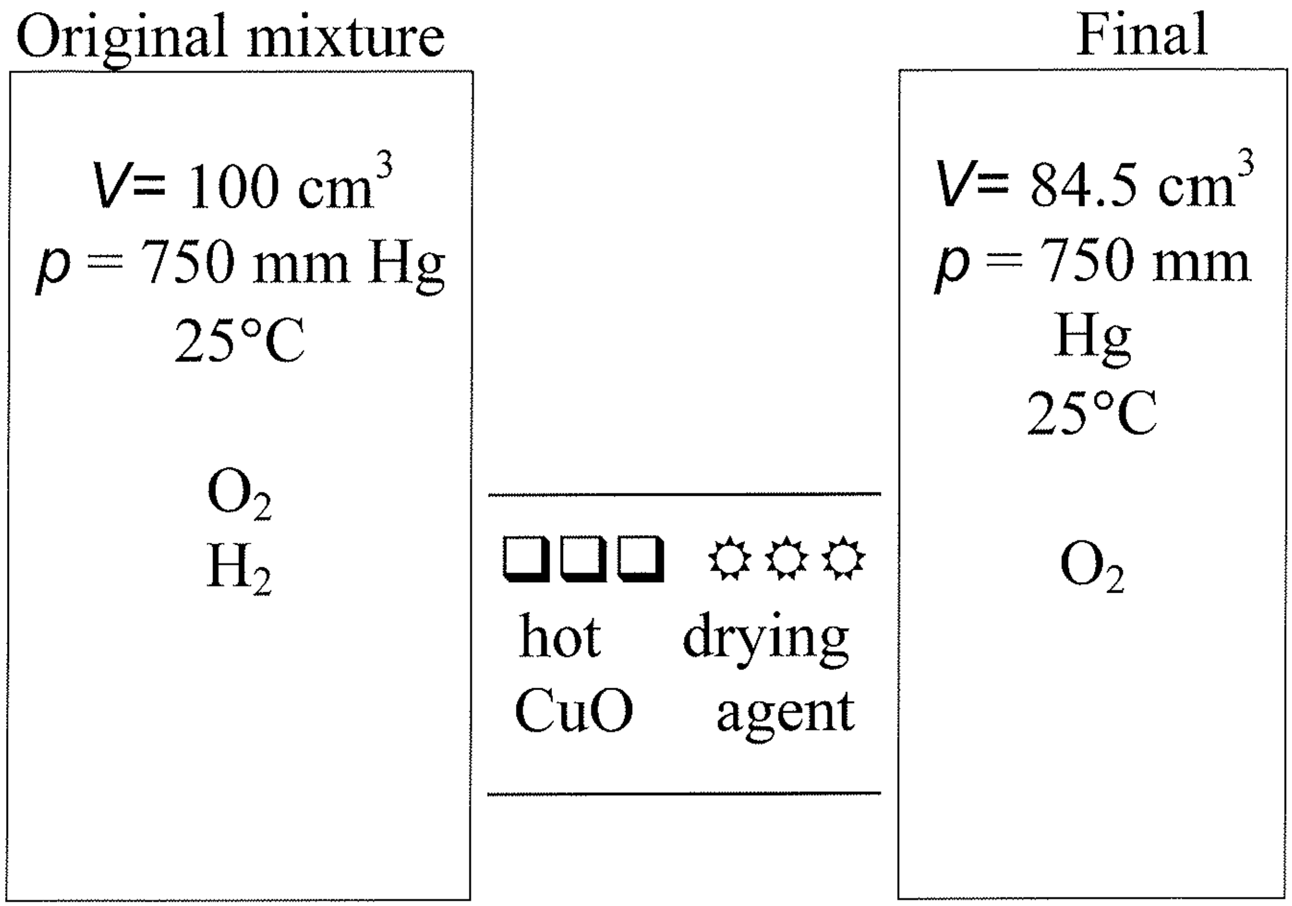
$x_{N2} = 0.98 = n_{N2} / [\ n_{N2} + n_{H2O}]$

$= n_{N2} / [\ n_{N2} + (0.150/18.0)]$

EXAMPLE

Problem 4. A mixture of oxygen and hydrogen is analyzed by passing it over hot copper oxide and through a drying tube. Hydrogen reduces the CuO to metallic Cu. Oxygen then reoxidizes the copper back to CuO. 100 cm^3 of the mixture measured at 25°C and 750 mm yields 84.5 cm^3 of dry oxygen measured at 25°C and 750 mm after passage over CuO and the drying agent. What is the original composition of the mixture? {Hint: First write balanced chemical equations for the reactions.}

Draw a picture



Question: In original mixture, $x_{O2} = ?$ $x_{H2} = ?$

Principles and Definitions:

Chemical reactions:

$O_2 + CuO \rightarrow$ no reaction

$H_2 + CuO \rightarrow Cu + H_2O$

amount of Cu formed is stoichiometric - based on moles H_2 reacted.

$\frac{1}{2}O_2 + Cu \rightarrow CuO$

amount of O_2 removed is stoichiometric - based on moles Cu present, which in turn, is the same as moles H_2 reacted.

Definition of mole fraction:

$$x_{O2} = n_{O2} / (n_{O2} + n_{H2})$$

Assume ideal gas behavior: $pV = nRT$

Solution:

Let n = original no. of moles of gas = $n_{O2} + n_{H2}$

$n = pV/RT$

$= 750$ mm Hg × (1 atm/760 mm Hg)
× 100 cm^3 L × (1 L/10^3 cm^3)
÷ {8.20578×10^{-2} L atm K^{-1} mol^{-1}
× (25+273.15) K}

= 4.034×10^{-3} moles gas

$x_{H2} + x_{O2} = 1$

moles O_2 in original = x_{O2} (4.034×10^{-3} moles)

moles H_2 in original = x_{H2} (4.034×10^{-3} moles)

After reaction, no. of moles of O_2 gas = pV/RT

= 750 mm Hg × (1 atm/760 mm Hg)
× 84.5 cm^3 L× (1 L/10^3 cm^3)
÷ {8.20578×10^{-2} L atm K^{-1} mol^{-1}
× (25+273.15) K}

= 3.408×10^{-3} moles O_2 gas are left

Let y = moles of H_2 reacted

$$H_2 + CuO \rightarrow Cu + H_2O \qquad (1)$$

$$y \qquad y \qquad y \qquad y \text{ moles}$$

This amount of Cu then reacts, using up ½ y moles of O_2:

$$\tfrac{1}{2}O_2 + Cu \rightarrow CuO \qquad (2)$$

$$\tfrac{1}{2}y \qquad y \qquad y$$

From reaction (1),

moles of $H_2 = n_{H2} = x_{H2}\,(4.034\times10^{-3} \text{ moles}) = y$

or $x_{H2} = y\,/\,(4.034\times10^{-3} \text{ moles})$

Because of reaction (2),

moles of O_2 left $= x_{O2}\,(4.034\times10^{-3} \text{ moles}) - \tfrac{1}{2}y$

$=3.408\times10^{-3}$moles

or $x_{O2} = [3.408\times10^{-3} + \tfrac{1}{2}y]\,/\,(4.034\times10^{-3})$

Since $x_{H2} + x_{O2} = 1$

$\therefore\ y\,/\,(4.034\times10^{-3} \text{ moles})$

$+ [3.408\times10^{-3} + \tfrac{1}{2}y]\,/\,(4.034\times10^{-3}) = 1$

or $(3/2)\,y = (4.034 - 3.408)\times10^{-3}$

$y = 4.173\times10^{-4}$ moles

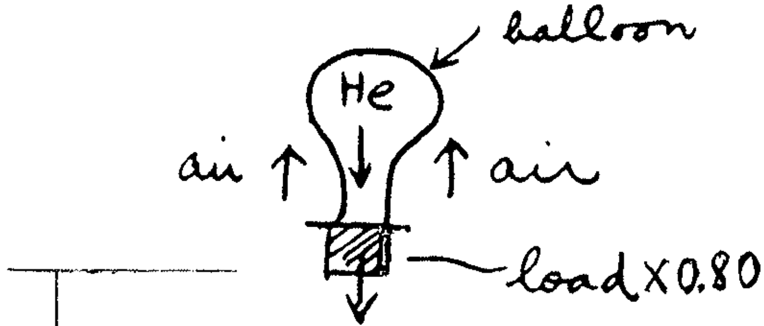
Substitute into $x_{H2} = y\,/\,(4.034\times10^{-3} \text{ moles})$ to get $x_{H2} = 0.103$ **Answers**

$\therefore x_{O2} = (1 - x_{H2}) = 0.897$

EXAMPLE

Problem 5. A balloon having a capacity of 10,000 m^3 is filled with helium at 20°C and 1 atm pressure. If the balloon is loaded with 80% of the load that it can lift at ground level, at what height will the balloon come to rest? Assume that the volume of the balloon is constant, the atmosphere isothermal, 20°C; the molecular weight of air is 28.8 and the ground level pressure is 1 atm. The mass of the balloon is 1.3×10^6 g.

Draw a picture

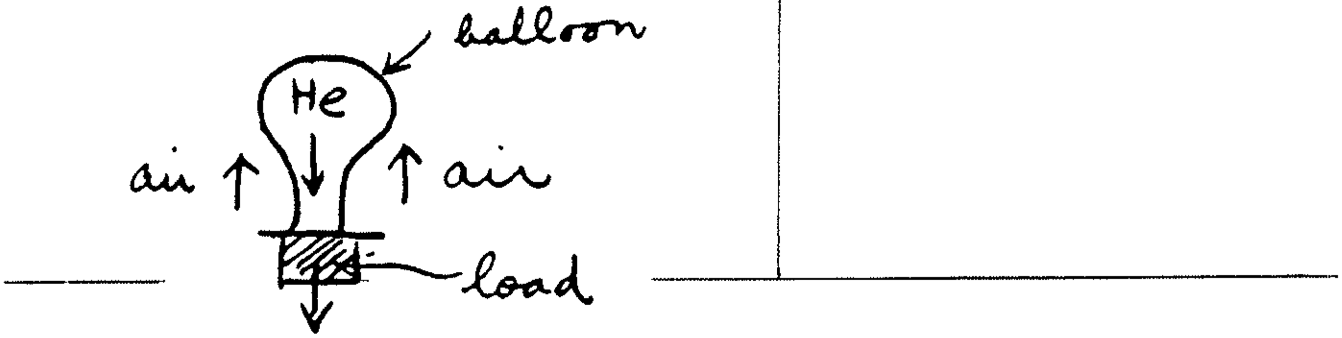


At $z = h = ?$

$p = ?$ atm

$V = 10^4\ m^3$ He

$T = (20 + 273.15)$ K



At ground level ($z = 0$)

Atmosphere : $p = 1$ atm

$V = 10^4\ m^3$ He

$T = (20 + 273.15)$ K

mass of empty balloon = 1.3×10^6 g

At equilibrium on the ground:

Total mass = $m_{\text{balloon}} + m_{\text{load}} + m_{\text{He}}$

To achieve equilibrium at $z = h$:

Total mass= $m_{\text{balloon}} + 0.80(m_{\text{load}}) + m_{\text{He}}$

Question: $h = ?$

Principles and Definitions:

1. Archimedes principle : At equilibrium, the surrounding fluid (air, in this case) supports a body (balloon + load it is carrying) whose weight is equal to the weight of the displaced fluid.

This means:

at height $z = h$,

equilibrium is reached when mass of displaced air = $m_{\text{balloon}} + 0.80\ (m_{\text{load}}) + m_{\text{He}}$

at ground level $z = 0$,

equilibrium is reached when mass of displaced air = $m_{\text{balloon}} + m_{\text{load}} + m_{\text{He}}$

2. Barometric formula:

$p/p_0 = \rho/\rho_0 = \exp[-Mgh/RT]$

3. Assume ideal gas behavior for both air and He: $pV = nRT$

units : $\text{J} \equiv \text{kg m}^2\ \text{s}^{-2}$

Assume that we can neglect the volume of air displaced by the load in comparison to the volume of the balloon.

Solution:

At ground level,

the mass of displaced air = $\rho_0 V$

At h, the mass of displaced air = ρV

The equilibrium conditions are:

$$m_{\text{balloon}} + m_{\text{load}} + m_{\text{He}} = \rho_0 V \quad (1) \quad \text{and}$$

$$m_{\text{balloon}} + 0.80(m_{\text{load}}) + m_{\text{He}} = \rho V \quad (2)$$

Eq. (2) ÷Eq. (1):

$$\rho/\rho_0 = [\, m_{\text{balloon}} + 0.80(m_{\text{load}}) + m_{\text{He}}]/[\, m_{\text{balloon}} + m_{\text{load}} + m_{\text{He}}] \quad (3)$$

$$m_{\text{He}} = \mathrm{M_{He}}(pV/RT)$$

$$= 4.0 \text{ g mol}^{-1} \times 1 \text{ atm} \times 10^4 \text{ m}^3 \times (10^3 \text{ L} / 1 \text{ m}^3) \div [\, 8.20578 \times 10^{-2} \text{ L atm K}^{-1} \text{ mol}^{-1} \times 293.15 \text{ K}]$$

$$= 1.663 \times 10^6 \text{ g}$$

$$\rho_0 V = m_{\text{air}}$$

$$= 28.8 \text{ g mol}^{-1} \times 1 \text{ atm} \times 10^4 \text{ m}^3 \times (10^3 \text{ L} / 1 \text{ m}^3) \div [\, 8.20578 \times 10^{-2} \text{ L atm K}^{-1} \text{ mol}^{-1} \times 293.15 \text{ K}]$$

$$= 1.1972 \times 10^7 \text{ g}$$

From eq. (1), we obtain

$$m_{\text{load}} = 1.1972 \times 10^7 \text{ g} - [1.3 \times 10^6 \text{ g} + 1.663 \times 10^6 \text{ g}]$$

$$= 9.01 \times 10^6 \text{ g}$$

Substitute this into eq. (3),

$$\rho/\rho_0$$

$$= [1.3\times10^6 + 1.663\times10^6 + 0.80\times9.01\times10^6] \div [1.3\times10^6 + 1.663\times10^6 + 9.01\times10^6]$$

$$= 0.85$$

$$\rho/\rho_0 = 0.85 = \exp[-Mgh/RT]$$

$$= \exp[28.8 \text{ g mol}^{-1} \times 1 \text{ kg}/10^3 \text{ g} \times 9.80665 \text{ m s}^{-2} \times h \text{ m} \div 8.31451 \text{ J K}^{-1} \text{ mol}^{-1} \times 293.15 \text{ K}]$$

$$0.85 = \exp[1.159\times10^{-4} h]$$

or $1.159\times10^{-4} h = \ln(0.85)$,

$h = 1.402\times10^3$ m **Answer**

Approximate method:

If $m_{\text{balloon}} + m_{\text{He}} << m_{\text{load}}$ then, the equilibrium conditions are:

$$m_{\text{load}} \approx \rho_0 V \qquad 0.80(m_{\text{load}}) \approx \rho V$$

$$\rho/\rho_0 \approx 0.80 = \exp[-Mgh/RT]$$

$$= \exp[28.8 \text{ g mol}^{-1} \times 1 \text{ kg}/10^3 \text{ g} \times 9.80665 \text{ m s}^{-2} \times h \text{ m} \div 8.31451 \text{ J K}^{-1} \text{ mol}^{-1} \times 293.15 \text{ K}]$$

$$0.80 \approx \exp[1.159\times10^{-4} h]$$

or $1.159\times10^{-4} h \approx \ln(0.80)$,

$\therefore h \approx 1.926\times10^3$ m not bad