Solutions to Problem Set 2

1. isothermal, reversible

3 moles ideal gas 20.0 L, 300 K → 60.0 L, 300 K

Equation	Basis for the equation	Eq. #
$d\mathbf{U} = (\partial \mathbf{U}/\partial T)_V dT + (\partial \mathbf{U}/\partial V)_T dV$	.$\mathbf{U} = \mathbf{U}(T,V)$	1
$dT = 0$	.Given	2
$(\partial \mathbf{U}/\partial V)_T = 0$	.Given an ideal gas, property of an ideal gas	3
$\Delta \mathbf{U} = \int d\mathbf{U} = 0$ **Answer**	.	4
$d\mathbf{H} = (\partial \mathbf{H}/\partial T)_p dT + (\partial \mathbf{H}/\partial p)_T dp$	.$\mathbf{H} = \mathbf{H}(T,p)$	5
$dT = 0$	.Given	
$(\partial \mathbf{H}/\partial p)_T = 0$	. property of an ideal gas	6
$\Delta \mathbf{H} = \int d\mathbf{H} = 0$ **Answer**	.	7
$\delta W = - p_{op}\, dV$	Definition	8
$p_{op} = p_{gas}$	.reversible expansion	9
$p_{gas} = nRT/V$	.ideal gas equation of state	10
$W = - \int (nRT/V)dV$ $= - nRT \ln(V_f/Vi)$ $= - 3\text{mol}\ (8.31451\ \text{J mol}^{-1}\ \text{K}^{-1})$ $\bullet\ 300\ \text{K} \ln(60.0/20.0)$ $W = - 8220.\ \text{J}$ **Answer**	.Using Eq 8,9,10 in and integrating between given initial and final conditions	11 12 13
$\Delta \mathbf{U} = q + W$	.First Law of thermodynamics	14
$q = -W = + 8220.\ \text{J}$ **Answer**	.Applying Eq 4, 14 and 13	

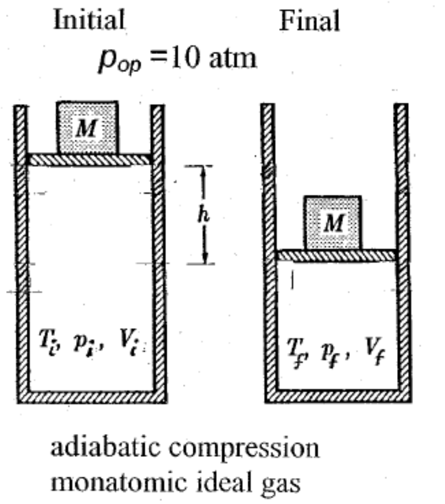
2. isothermal, reversible

1 mole van der Waals gas 10.0 L, 300 K → 30.0 L, 300 K

Equation	Basis for the equation	Eq. #
$d\mathbf{U} = (\partial \mathbf{U}/\partial T)_V dT + (\partial \mathbf{U}/\partial V)_T dV$	.$\mathbf{U} = \mathbf{U}(T,V)$	1
$dT = 0$	.Given	2
$p = RT/(V-b) – a/V^2$	van der Waals Equation of state, V is molar volume	3
$(\partial \mathbf{U}/\partial V)_T = T(\partial p/\partial T)_V - p$	derived from first and second law, as we shall see derived later in lecture notes part 4	4
$(\partial p/\partial T)_V = (\partial\ /\partial T)_V\{ RT/(V-b) – a/V^2\}$	Apply to a van der Waals gas	5
$(\partial p/\partial T)_V = R/(V-b)$	Differentiating, provided a is not a function of T, as is the case in this problem	6
$(\partial \mathbf{U}/\partial V)_T = T(\partial p/\partial T)_V – p$ $= RT/(V-b) - \{ RT/(V-b) – a/V^2\}$ $= a/V^2$		7

$\Delta\mathbf{U} = \int(\partial\mathbf{U}/\partial V)_T dV = a\int dV/V^2$ $= -a[V_f^{-1} - V_i^{-1}]$ $= -5.49\ L^2 atm\ mol^{-2}\ [30^{-1} - 10^{-1}]$ $= +0.366\ L\ atm\ mol^{-1}$ **Answer**	.Substituting Eq 7 and Eq 2 into Eq 1 and integrating given $a = 5.49\ L^2 atm\ mol^{-2}$	8
$\delta W = -p_{op}\ dV$	Definition	9
$p_{op} = p_{gas}$	.reversible expansion	10
$p_{gas} = RT/(V-b) - a/V^2$	van der Waals equation of state from Eq 3	
$W = -\int (RT/(V-b))dV + a\int dV/V^2$ $= -RT\ \ln\{(V_f-b)/(V_i-b)\} - a[V_f^{-1} - V_i^{-1}]$ $= -(0.0820578\ L\ atm\ mol^{-1}\ K^{-1})$ $\bullet\ 300\ K\ \ln\{(30.0-0.064)/(10.0-0.064)\}$ $+ 5.49\ L^2 atm\ mol^{-2}\ [30^{-1} - 10^{-1}]$ $W = -27.15 + 0.366 = -26.78\ L\ atm\ mol^{-1}$ **Answer**	.Using Eq 8,9,10 in and integrating between given initial and final conditions Given $b = 0.064\ L\ mol^{-1}$, $a = 5.49\ L^2 atm\ mol^{-2}$	11 12 13
$\Delta\mathbf{U} = q + W$	.First Law of thermodynamics	14
$q = \Delta\mathbf{U} - W = 0.366 + 26.78 = 27.15$ **Answer**	.Applying Eq 4, 14 and 13	15
$d\mathbf{H} = (\partial\mathbf{H}/\partial T)_p dT + (\partial\mathbf{H}/\partial p)_T dp$	.**H** = **H**(T,p)	
$dT = 0$	.Given	
$(\partial\mathbf{H}/\partial p)_T = -T(\partial V/\partial T)_p + V$	derived from first and second law see lecture notes part 4 for derivation. We could use this method but to get $(\partial V/\partial T)_p$ we will have to express V in terms of p from the van der Waals equation of state. This is too time consuming to do.	
$\Delta\mathbf{H} = \Delta\mathbf{U} + \Delta(pV)$	Instead we use **H** = **U** +pV, by definition	16
$p = RT/(V-b) - a/V^2$ $p_i = (0.0820578\ L\ atm\ mol^{-1}\ K^{-1})$ $\bullet 300/(10-0.064) - 5.49/10^2 = 2.42\ atm$ $p_f = (0.0820578\ L\ atm\ mol^{-1}\ K^{-1})$ $\bullet 300/(30-0.064) - 5.49/30^2 = 0.816 atm$	Using the equation of state to find p_i and p_f from known T and V	
$\Delta\mathbf{H} = 0.366 + [0.816 \bullet 30.0 - 2.42 \bullet 10.0]$ $= 0.646\ L\ atm\ mol^{-1}$ **Answer**		

3.



n = 1 mole	n = 1 mole
p_i = 1 atm	p_f = 10 atm
T_i = 27+273.15	T_f =
V_i=?	V_f=?

Equation	Basis for the equation	Eq. #
$\delta W = - p_{op}dV$	General definition of work of compression-expansion	1
$W = -p_{op}\int dV = -10.0\ \text{atm}\bullet\ [V_f-V_i]$	Given, p_op = constant = 10.0 atm Integrating Eq 1	2
$pV = nRT$	Ideal gas equation of state	3
$V_f = RT_f/10.0 \quad V_i = R300/1.0$	Using given data and Eq 3	4
$W = -10.0\ \text{atm}\bullet\ [RT_f/10.0 - R300/1.0]$	From Eq 2 and 4	5
$d\mathbf{U} = (\partial \mathbf{U}/\partial T)_V dT + (\partial \mathbf{U}/\partial V)_T dV$	General, from $\mathbf{U} = \mathbf{U}(T,V)$	6
$d\mathbf{U} = C_V dT + (\partial \mathbf{U}/\partial V)_T dV$	General, using definition of C_V	7
$(\partial \mathbf{U}/\partial V)_T = 0$	Property of an ideal gas	8
$d\mathbf{U} = \delta q + \delta W$	First law of thermodynamics	9
$\delta q = 0,\ q = 0$ **Answer**	For an adiabatic process, as in this problem	10
$\Delta\mathbf{U} = W + 0$	From Eq 9 and 10	11
	Case I monatomic ideal gas, $C_V = (3/2)R$	
$\Delta\mathbf{U} = \int C_V dT = C_V[T_f-T_i] = (3/2)R[T_f-300]$	from Eq 7 and C_V independent of temperature and = 3/2 R for monatomic ideal gas, given T_i=300	12
$(3/2)R[T_f-300] = -10.0\ \text{atm}\bullet\ [RT_f/10.0 - R300/1.0]$	From Eq 12 and 11 and 5	13
$T_f = 1380$ K **Answer**	Solving Eq 13 for T_f	14
$W = +13.469$ kJ **Answer**	From substituting Eq 14 into Eq 5	15
$\Delta\mathbf{U} = +13.469$ kJ **Answer**	From Eq 11 and Eq 15	16
$d\mathbf{H} = (\partial \mathbf{H}/\partial T)_p dT + (\partial \mathbf{H}/\partial p)_T dp$	.$\mathbf{H} = \mathbf{H}(T,p)$	17

$d\mathbf{H} = C_p dT + (\partial \mathbf{H}/\partial p)_T dp$	.Using Eq 17 and definition of C_p	18
$(\partial \mathbf{H}/\partial p)_T dp = 0$	Property of an ideal gas	19
$C_p = C_V + R = (5/2)R$	Property of an ideal gas which is monatomic	20
$\Delta \mathbf{H} = \int C_p dT = (5/2)R[T_f - T_i]$ $= (5/2)(8.31451 \text{ J mol}^{-1} \text{ K}^{-1})\ [1380-300]$ $= + 20.782$ kJ **Answer**	Using Eq 18, 19, 20 and integrating	21
Case II	diatomic ideal gas, $C_V = (5/2)R$	22
$(5/2)R[T_f-300] = -10.0$ atm• $[RT_f/10.0 - R300/1.0]$	Using Eq 13 and 22 for a diatomic ideal gas	23
$T_f = 1071.4$ K **Answer**	Solving for T_f	24
$W = +16.035$ kJ **Answer**	Substituting T_f into Eq 5	25
$\Delta \mathbf{U} = +16.035$ kJ **Answer**	Replacing C_V by (5/2)R and T_f into Eq 13	26
$\Delta \mathbf{H} = +22.448$ kJ **Answer**	Replacing Cp by (7/2)R and T_f into Eq 23	27
IF n moles instead of n=1 mole		
T_f no change **Answer**	Multiply both sides of Eq 13 or 23 by n	
W = multiply by n **Answer**	Extensive property	
$\Delta \mathbf{U}$ = multiply by n **Answer**	Extensive property	
$\Delta \mathbf{H}$ = multiply by n **Answer**	Extensive property	

4. For problems 4 and 6 which are for ideal gases undergoing a reversible adiabatic process, we will start from the general equations and apply these three very specific conditions (ideal gas, adiabatic process, reversible process) to it so that we may end up with equations that we can use for both Problem 4 and Problem 6.

Equation	Basis for the equation	Eq. #
$d\mathbf{U} = \delta q + \delta W$	First law of thermodynamics	1
$\delta W = - p_{op} dV$	Definition of compression-expansion work	2
$d\mathbf{U} = (\partial \mathbf{U}/\partial T)_V dT + (\partial \mathbf{U}/\partial V)_T dV$	General, from $\mathbf{U} = \mathbf{U}(T,V)$	3
$d\mathbf{U} = C_V dT + (\partial \mathbf{U}/\partial V)_T dV$	Introduce definition of C_v into Eq 3	4
$\delta q - p_{op} dV = C_V dT + (\partial \mathbf{U}/\partial V)_T dV$	From Eq 1, 2, and 4	5
$(\partial \mathbf{U}/\partial V)_T = 0$	Property of an ideal gas	6
$\delta q = 0$	For an adiabatic process	7
$- p_{op} dV = C_V dT$	Substituting Eq 6 and 7 into Eq 5	8
$p_{op} = p_{gas}$	For a reversible process the opposing pressure must match p_{gas} at all times	9
$p_{gas} V = nRT$	Equation of state for an ideal gas	10
$-RTdV/V = C_V dT$	Using Eq 9 and 10 in Eq 8 for one mole	11
$- dV/V = (C_V/R)\ dT/T$	Rearranging	12
$- \ln (V_f/V_i) = (C_V/R)\ dT/T$	Since C_V is independent of temperature and integrating	13
$(T_f/T_i) = (V_i/V_f)^{(CV/R)}$	Using the definition of ln This form relates initial and final T and V	14

$(T_f/T_i) = p_fV_f/p_iV_i$ $p_fV_f/p_iV_i = (V_i/V_f)^{(CV/R)}$ $p_f/p_i = (V_f/V_i)^{-1}(V_i/V_f)^{(CV/R)}$ $p_f/p_i = (V_f/V_i)^{-1-R/CV} = (V_f/V_i)^{(-CV-R)/CV}$ $p_f/p_i = (V_f/V_i)^{(-Cp/CV)}$ $V_i/V_f = [p_f/p_i]^{(CV/Cp)}$ $V_f/V_i = [p_f/p_i]^{-(CV/Cp)}$	There are several ways of rearranging Eq 14 into other forms, by using the ideal gas law and using $C_p=C_V+R$ This relates initial and final p and V	15 16
$T_f/T_i = [p_f/p_i]\bullet[V_f/V_i]$ $= [p_f/p_i]\bullet [p_f/p_i]^{-(CV/Cp)}$ $= [p_f/p_i]^{(Cp-CV)/Cp}$ $T_f/T_i = [p_f/p_i]^{R/Cp}$	We find yet another form This relates initial and final p and T These equations apply *only to adiabatic reversible processes for an ideal gas*	17 19

We will need equation 19 for problems 4 and 6 which involve adiabatic reversible ideal gas, the assumptions we used in these derivations.

4. adiabatic, reversible

1 mole ideal gas 1.0 atm, 300 K → 10.0 atm, T_f

Equation	Basis for the equation	Eq. #
$T_f/T_i = [p_f/p_i]^{R/Cp}$ $T_f/300 = [10.0/1.0]^{R/Cp}$ $T_f/300 = [10.0/1.0]^{2/5}$ $T_f = 300\ 10^{2/5} = 754$ K	use Eq 19 derived above For an ideal monatomic gas, $C_p = 5/2\ R$ Solve for T_f	1
$q = 0$ **Answer**	adiabatic	2
	We choose not to calculate W directly because it is easier to get this value from ΔU	
$d\mathbf{U} = (\partial \mathbf{U}/\partial T)_V dT + (\partial \mathbf{U}/\partial V)_T dV$	General, from $\mathbf{U} = \mathbf{U}(T,V)$	3
$d\mathbf{U} = C_V dT + (\partial \mathbf{U}/\partial V)_T dV$	General, using definition of C_V	4
$(\partial \mathbf{U}/\partial V)_T = 0$	Property of an ideal gas	5
$\Delta\mathbf{U} = \int C_V dT = (3/2)R[T_f-T_i] =$ $= 1.5(8.31451\ J\ mol^{-1}K^{-}1)[754-300]$ $\Delta\mathbf{U} = 5662\ J\ mol^{-1}$ **Answer**		6 7
$d\mathbf{U} = \delta q + \delta W$ $q = 0$ $\Delta\mathbf{U} = W$ $W = 5662\ J\ mol^{-1}$ **Answer**	First law of thermodynamics For an adiabatic process, as in this problem	8 9 10 11
$d\mathbf{H} = (\partial \mathbf{H}/\partial T)_p dT + (\partial \mathbf{H}/\partial p)_T dp$	.$\mathbf{H} = \mathbf{H}(T,p)$	12
$d\mathbf{H} = C_p dT + (\partial \mathbf{H}/\partial p)_T dp$	.Using Eq 12 and definition of C_p	13
$(\partial \mathbf{H}/\partial p)_T dp = 0$	Property of an ideal gas	14

$C_p = C_V + R = (5/2)R$	Property of an ideal gas which is monatomic	15
$\Delta \mathbf{H} = \int C_p dT = (5/2)R[T_f - T_i]$ $= (5/2)(8.31451\ \text{J mol}^{-1}\ \text{K}^{-1})\ [754-300]$ $= +\ 9437\ \text{J mol}^{-1}$ **Answer**	Using Eq 13, 14, 15 and integrating	16

5. adiabatic, constant p_{op}= 1.0 atm

1 mole ideal gas 10 atm, 300 K → 1.0 atm, T_f

This is very similar to Problem 3

Equation	Basis for the equation	Eq. #
$\delta W = -\ p_{op} dV$	General definition of work of compression-expansion	1
$W = -p_{op}\int dV = -1.0\ \text{atm}\bullet [V_f - V_i]$	Given, p_{op} = constant = 1.0 atm Integrating Eq 1	2
$pV = nRT$	Ideal gas equation of state	3
$V_f = RT_f/1.0 \quad V_i = R300/10.0$	Using given data and Eq 3	4
$W = -1.0\ \text{atm}\bullet [RT_f/1.0 - R300/10.0]$	From Eq 2 and 4	5
$d\mathbf{U} = (\partial \mathbf{U}/\partial T)_V dT + (\partial \mathbf{U}/\partial V)_T dV$	General, from **U** = **U**(T,V)	6
$d\mathbf{U} = C_V dT + (\partial \mathbf{U}/\partial V)_T dV$	General, using definition of C_V	7
$(\partial \mathbf{U}/\partial V)_T = 0$	Property of an ideal gas	8
$d\mathbf{U} = \delta q + \delta W$	First law of thermodynamics	9
$\delta q = 0,\ q = 0$ **Answer**	For an adiabatic process, as in this problem	10
$\Delta \mathbf{U} = W + 0$	From Eq 9 and 10	11
	Case I monatomic ideal gas, $C_V = (3/2)R$	
$\Delta \mathbf{U} = \int C_V dT = C_V[T_f - T_i] = (3/2)R[T_f - 300]$	from Eq 7 and C_V independent of temperature and = 3/2 R for monatomic ideal gas, given T_i=300	12
$(3/2)R[T_f - 300]$ $= -1.0\ \text{atm}\bullet[RT_f/1.0 - R300/10.0]$	From Eq 12 and 11 and 5	13
$T_f = 192$ K **Answer**	Solving Eq 13 for T_f	14
$W = -1347$ J **Answer**	From substituting Eq 14 into Eq 5	15
$\Delta \mathbf{U} = -1347$ J **Answer**	From Eq 11 and Eq 15	16
$d\mathbf{H} = (\partial \mathbf{H}/\partial T)_p dT + (\partial \mathbf{H}/\partial p)_T dp$	.**H** = **H**(T,p)	17
$d\mathbf{H} = C_p dT + (\partial \mathbf{H}/\partial p)_T dp$	.Using Eq 17 and definition of C_p	18
$(\partial \mathbf{H}/\partial p)_T dp = 0$	Property of an ideal gas	19
$C_p = C_V + R = (5/2)R$	Property of an ideal gas which is monatomic	20
$\Delta \mathbf{H} = \int C_p dT = (5/2)R[T_f - T_i]$ $= (5/2)(8.31451\ \text{J mol}^{-1}\ \text{K}^{-1})\ [192-300]$ $= -\ 2244.8$ J **Answer**	Using Eq 18, 19, 20 and integrating	21
Case II	$C_V = (5/2)R$ as in a diatomic ideal gas	22
$(5/2)R[T_f - 300] = -1.0\ \text{atm}\bullet$ $[RT_f/1.0 - R300/10.0]$	Using Eq 13 and 22 for a diatomic ideal gas	23
$T_f = 222.9$ K **Answer**	Solving for T_f	24
$W = -1603.5$ J **Answer**	Substituting T_f into Eq 5	25

$\Delta \mathbf{U}$ = -1603.5 J **Answer**	Replacing C_V by (5/2)R and T_f into Eq 13	26
$\Delta \mathbf{H}$ = -2244.8 J **Answer**	Replacing C_p by (7/2)R and T_f into Eq 23	27

6. adiabatic, reversible

1 mole ideal gas 10.0 atm, 300 K → 1.0 atm, T_f

Similar to problem 4

Equation	Basis for the equation	Eq. #
$T_f/T_i = [p_f/p_i]^{R/Cp}$ $T_f/300 = [1.0/10.0]^{R/Cp}$ $T_f/300 = [1.0/10.0]^{2/5}$ $T_f = 300\ (0.1)^{2/5} = 119.4$ K	use Eq derived previously for prob 4 For an ideal monatomic gas, C_p = 5/2 R Solve for T_f	1
q = 0 **Answer**	adiabatic	2
	We choose not to calculate W directly because it is easier to get this value from $\Delta \mathbf{U}$	
$d\mathbf{U} = (\partial \mathbf{U}/\partial T)_V dT + (\partial \mathbf{U}/\partial V)_T dV$	General, from $\mathbf{U} = \mathbf{U}(T,V)$	3
$d\mathbf{U} = C_V dT + (\partial \mathbf{U}/\partial V)_T dV$	General, using definition of C_V	4
$(\partial \mathbf{U}/\partial V)_T = 0$	Property of an ideal gas	5
$\Delta \mathbf{U} = \int C_V dT = (3/2)R[T_f - T_i] =$ $= 1.5(8.31451\ \text{J mol}^{-1}\text{K}^{-}1)$ $\bullet[119.4-300]$ $\Delta \mathbf{U}$ = - 2252 J mol^{-1} **Answer**		6 7
$d\mathbf{U} = \delta q + \delta W$ q = 0 $\Delta \mathbf{U}$ = W W = -2252 J mol^{-1} **Answer**	First law of thermodynamics For an adiabatic process, as in this problem	8 9 10 11
$d\mathbf{H} = (\partial \mathbf{H}/\partial T)_p dT + (\partial \mathbf{H}/\partial p)_T dp$	.$\mathbf{H} = \mathbf{H}(T,p)$	12
$d\mathbf{H} = C_p dT + (\partial \mathbf{H}/\partial p)_T dp$	.Using Eq 12 and definition of C_p	13
$(\partial \mathbf{H}/\partial p)_T dp = 0$	Property of an ideal gas	14
$C_p = C_V + R = (5/2)R$	Property of an ideal gas which is monatomic	15
$\Delta \mathbf{H} = \int C_p dT = (5/2)R[T_f - T_i]$ $= (5/2)(8.31451\ \text{J mol}^{-1}\ \text{K}^{-1})$ $\bullet\ [119.4-300]$ $\Delta \mathbf{H}$ = = - 3754 J mol^{-1} **Answer**	Using Eq 13, 14, 15 and integrating	16

7. p = constant = 1 atm ; warm the air in the house from T_1 to T_2 , air being expelled through keyholes etc as the air expands.

Equation	Basis for the equation	Eq. #
$\delta q = nC_p dT$	relation between heat and heat capacity at constant pressure	1
$q = \int nC_p dT$ $= C_p \int n dT$ $n = pV/RT$ $q = C_p \int (pV/RT) dT$ $= C_p (pV/R) \int dT/T$ $q = C_p (pV/R) \ln T_2/T_1$ **Q.E.D.**	Assume that air has a C_p independent of temperature. What is unusual about this problem is that n is varying during the heating of the air. Assume that air behaves ideally, and the volume of the house is V.	2 3 4 5 6
$q = (7/2)R\ (1.0\text{ atm})(2000\times10^3\text{L})/R \bullet \ln(293/288)$ $= (7/2)\ 2\times10^6 \ln(293/288)$ L atm $q = 120477$ L atm = 12207 kJ **Answer**	10^3 L = 1 m^3 given $C_V = (5/2)R$; $C_p = (7/2)R$	7

8. p = constant = 1 atm ; cool the air in the house from T_1 to T_2 , air leaking in from outside as the temperature goes down

Equation	Basis for the equation	Eq. #
$n_1 = pV/RT_1$ $n = n_1 + n_{out}$ $pV/RT = n$ $n_{out} = n - n_1 = pV/RT - pV/RT_1$	n_{in} moles of air inside the house, originally at T_1, n_{out} moles of air coming in from the outside originally at T_0 ; all have to be cooled down to temperature T_2. n is changing with T, as air is coming in during the cooling process	1 2 3 4
$\delta q = nC_p dT$		5
$q = \int nC_p dT = C_p \int n dT$ $q = q_1 + q_2$	Definition; C_p is independent of T total heat extracted = heat from original air (q_1) and heat from outside air (q_2)	6 7
$q_1 = C_p \int_{T1}^{T2} n_1 dT = C_p(pV/RT_1)\int_{T1}^{T} dT$ $= C_p(pV/RT_1)[T_2 - T_1]$	Substituting Eq 1 into Eq 6	8 9
$q_2 = C_p \int_{T0}^{T2} (n - n_1) dT$ $= C_p \int_{T0}^{T2} (n - n_1) dT$ $= C_p \int_{T0}^{T2} \{(pV/RT) - pV/RT_1\} dT$ $= C_p (pV/R) \bullet \{ \int_{T0}^{T2} dT/T - (1/T_1)\int_{T0}^{T2} dT)\}$ $= C_p (pV/R) \bullet \{\ln(T_2/T_0) - [T_2 - T_0]/T_1\}$	Substituting Eq 4 into Eq 6	10 11 12

$q = q_1+q_2 = C_p (pV/R)$ $\bullet\{(T_2-T_1)/T_1)+ \ln(T_2/T_0) – [T_2-T_0]/T_1\}$ $q = C_p(pV/R)\bullet\{ -1 + \ln(T_2/T_0) + (T_0/T_1)\}$ **Q.E.D.** Different from Prob 7 because the number of moles of air are different	Sum	
$q = (7/2)R((1.0 \text{ atm})(2000\times10^3\text{L})/R)$ $\bullet\{ -1 + \ln(293/308) + (308/298)\}$ $q = 3.5(2\times10^6)$ $\bullet\{ -1 + \ln(293/308) + (308/298)\}$ L atm q = -114610 L atm = -11613 kJ **Answer**	Substituting T_1 = 298, T_2 = 293, T_0 = 308 K and p=1 atm, V = 2000 m^3	

9. Friction in the piston is equivalent to a pressure contribution p_f

isothermal expansion

one mole ideal gas $p_1 \rightarrow p_2$

Equation	Basis for the equation	Eq. #
$W = - \int p_{op}dV$ without friction $W = - \int (p_{op} + p_f)dV$ with friction	Definition Gas expands against a greater opposing pressure because piston sticks a bit	1 2
$d\mathbf{U} = C_V dT + (\partial \mathbf{U}/\partial V)_T dV$ $(\partial \mathbf{U}/\partial V)_T = 0$ $dT = 0$ $\Delta \mathbf{U} = 0$	$\mathbf{U} = \mathbf{U}(V, T)$ For an ideal gas isothermal	3 4 5 6
$\Delta \mathbf{U} = q + W$	First law of thermodynamics	7
$q = - W$ $= + \int (p_{op} + p_f)dV$	Using Eq 6 and 7	8 9
Compression, on the other hand		
$W = - \int p_{op}dV$ without friction $W = - \int (p_{op} - p_f)dV$ with friction	pressure being applied appears less as far as gas is concerned	10
$q = - W = + \int (p_{op} - p_f)dV$		
Problem statement not particularly clear about what p_{op} was. Assume expanding against a constant pressure of p_2 from initially p_1 to final pressure of p_2, $W = - \int (p_{op} + p_f)dV = (p_2+pf)[V_2-V_1]$ Isothermal : $V_1 = RT/p_1$= R298/1 atm = (0.0820578)298/1= 24.45 L V_2 = R298/0.5 = 48.90 L W = - (0.5+0.1)[24.45-48.90] = +14.67 L atm		
q = -14.67 L atm gas had to do extra work to overcome friction in piston		

10. **A** **B** **C**

1 mole ideal gas $p_1, T_1, V_1 \rightarrow p_2, T_2, V_2 \rightarrow p_3, T_3, V_3 \rightarrow p_4, T_4, V_4$

A reversible heating at constant p = 1 atm, from 298 to 373 K

B reversible isothermal expansion to $2V_2$

C reversible adiabatic cooling to 308 K

$C_p = (5/2)R$

Equation	Basis for the equation	Eq. #
$p_1 =$ 1 atm $T_1 = 298$ $V_1 = R298/1$ atm = 24.45 L	Given ideal gas	1 2
$p_2 =$ 1 atm $T_2 = 373$ $V_2 = R373/1$ atm = 30.61 L		3 4
$T_3 = T_2 = 373$ $V_3 = 2V2 = 61.22$ L $p_3 = R373/61.22 = 0.50$ atm		5 6
$T_4 = 308$ K p_4 , V_4 can be calculated but we actually do not need these		7
$d\mathbf{U} = C_V dT + (\partial \mathbf{U}/\partial V)_T dV$	$\mathbf{U} = \mathbf{U}(T,V)$ and definition of C_V	8
$(\partial \mathbf{U}/\partial V)_T = 0$ $d\mathbf{U} = C_V dT$	property of an ideal gas	9 10
$d\mathbf{H} = C_p dT + (\partial \mathbf{H}/\partial p)_T dp$	$\mathbf{H} = \mathbf{H}(T,p)$ and definition of C_p	11
$(\partial \mathbf{H}/\partial p)_T = 0$ $d\mathbf{H} = C_p dT$	property of an ideal gas	12 13
Step A $\Delta\mathbf{U} = (3/2)R(373-298) = 935.4$ J **Answer**	From Eq 10 and given $C_V = (3/2)R$	14
$\Delta\mathbf{H} = (5/2)R(373-298) = 1559.0$ J **Answer**	From Eq 13 and given $C_p = C_V + R = (5/2)R$	15
$W = - \int p_{op} dV = -1$ atm $[V_2 - V_1]$ $= -1 [30.61 - 24.45] = - 6.16$ L atm $= - 624.2$ J **Answer**	Definition and p = constant = 1 atm	16 17
$\Delta\mathbf{U} = q + W$ $q = \Delta\mathbf{U} - W = 935.4 + 624.2 = 1559.6$ J **Answer**	First law of thermodynamics	18 19
Step B $\Delta\mathbf{U} = 0$ J **Answer**	$dT = 0$	20
$\Delta\mathbf{H} = 0$ J **Answer**	$dT = 0$	21
$W = - \int p_{op} dV = - \int p_{gas} dV$ $W = - \int RT dV/V = - R373 \ln(V_3/V_2)$ $W = - R373 \ln 2 = -2149$ J **Answer**	$p_{op} = p_{gas}$ reversible expansion	22 23 24
$q = \Delta\mathbf{U} - W$ $q = 0 + 2149$ J **Answer**	First law of thermodynamics	25 26
Step C $\Delta\mathbf{U} = (3/2)R(308-373) = - 810.7$ J **Answer**	From Eq 10 and given $C_V = (3/2)R$	27

Δ**H** = (5/2)R(308-373)= - 1351.1 J **Answer**	From Eq 13 and given $C_p = C_V + R = (5/2)R$	
q = 0 **Answer**	adiabatic	
Δ**U** = q + W W = Δ**U** W = - 810.7 J **Answer**	First law of thermodynamics From Eq 27	
Overall Δ**U** = 124.7 J Δ**H** = 208.8 J W = -3583.9 q = 3708.6 J **Answer**		