## Solutions to Problem Set 5

1. ideal gas, $1 \mathrm{~mol} 1 \mathrm{~L} 25^{\circ} \mathrm{C} \rightarrow 100 \mathrm{~L} 25^{\circ} \mathrm{C}$

| Equation | Basis for the equation | Eq. \\| |
| :---: | :---: | :---: |
| $\mathrm{d} \mathbf{U}=(\partial \mathbf{U} / \partial \mathrm{T})_{\mathrm{V}} \mathrm{dT}+(\partial \mathbf{U} / \partial \mathrm{V})_{T} \mathrm{dV}$ | $\mathbf{U}=\mathbf{U}(\mathrm{T}, \mathrm{V})$ | 1 |
| $\mathrm{d} \mathbf{H}=(\partial \mathbf{H} / \partial \mathrm{T})_{\mathrm{p}} \mathrm{dT}+(\partial \mathbf{H} / \partial \mathrm{p})_{T} \mathrm{dp}$ | $\mathbf{H}=\mathbf{H}(\mathrm{T}, \mathrm{p})$ | 2 |
| $\Delta \mathbf{U}=0$ Answer <br> $\Delta \mathbf{H}=0$ Answer | For an ideal gas, $(\partial \mathbf{U} / \partial \mathrm{V})_{T}=0$ and $(\partial \mathbf{H} / \partial \mathrm{p})_{T}=0$ and $d T=0$ for this problem | $\begin{array}{\|l\|} \hline 3 \\ 4 \end{array}$ |
| $\mathrm{d} \mathbf{S}=(\partial \mathbf{S} / \partial \mathrm{T})_{\mathrm{V}} \mathrm{dT}+(\partial \mathbf{S} / \partial \mathrm{V})_{\mathrm{T}} \mathrm{d} \mathrm{V}$ | $\mathbf{S}=\mathbf{S}(\mathrm{T}, \mathrm{V})$ | 5 |
| $(\partial S / \partial V)_{T}=(\partial p / \partial T)_{V}$ | Derived (see lecture notes Part 4) starting from $\mathrm{d} \boldsymbol{U}=\Delta \mathrm{q}_{\mathrm{rev}}-p \mathrm{~d} V=T \mathrm{~d} \boldsymbol{S}-p \mathrm{~d} V$ and use of cross derivatives | 6 |
| $\begin{aligned} & p=R T / V \\ & (\partial \mathrm{p} / \partial \mathrm{T})_{\mathrm{V}}=\mathrm{R} / \mathrm{V} \\ & (\partial \mathrm{~S} / \partial \mathrm{V})_{T}=\mathrm{R} / \mathrm{V} \end{aligned}$ | Ideal gas equation of state for one mol Differentiating <br> Substituting Eq 8 into Eq 6 | $\begin{array}{\|l} \hline 7 \\ 8 \\ 9 \end{array}$ |
| $\Delta \mathbf{S}=\int(R / V) d V$ | Substituting Eq 9 into Eq 5 and using dT $=0$ for this problem | 10 |
| $\begin{aligned} & \Delta \mathbf{S}=\mathrm{R} \ln \left(\mathrm{~V}_{f} / \mathrm{V}_{\mathrm{i}}\right) \\ & =(1 \mathrm{~mol})\left(8.314 \mathrm{~J} \mathrm{~mol}^{-1} \mathrm{~K}^{-1}\right) \bullet \\ & \ln (100 / 1)=38.3 \mathrm{~J} \mathrm{~K}^{-1} \text { Answer } \end{aligned}$ | Integrating Eq 10 between the limits | 11 |
| $\mathbf{A}=\mathbf{U}-\mathrm{TS}$ | Definition | 12 |
| $\Delta \mathbf{A}=\Delta \mathbf{U}-\mathrm{T} \Delta \mathbf{S}$ | At constant T | 13 |
| $\begin{aligned} & \Delta \mathbf{A}=0-(298)\left(38.3 \mathrm{JK}^{-1}\right) \\ & \Delta \mathbf{A}=-11409 . \mathrm{J} \end{aligned}$ | From $\Delta \mathbf{U}=0$ Eq $3 \& \Delta \mathbf{S}$ from Eq 11, and given $T=$ 298 K | 14 |
| G = H-TS | Definition | 12 |
| $\Delta \mathbf{G}=\Delta \mathbf{H}-\mathrm{T} \Delta \mathbf{S}$ | At constant T | 13 |
| $\begin{aligned} & \Delta \mathbf{G}=0-(298)\left(38.3 \mathrm{JK}^{-1}\right) \\ & \Delta \mathbf{G}=-11409 . \mathrm{J} \end{aligned}$ | From $\Delta \mathbf{H}=0$ Eq $4 \& \Delta \mathbf{S}$ from Eq 11, and given $T=$ 298 K | 14 |

2. (a) ideal gas, $1 \mathrm{~mol} 22.4 \mathrm{~L}, 0^{\circ} \mathrm{C} \rightarrow 224 \mathrm{~L}, 0^{\circ} \mathrm{C}$ isothermal, reversible

| Equation | Basis for the equation | Eq. $\#$ |
| :---: | :---: | :---: |
| $\Delta \mathbf{U}=0 \quad$ Answer | Using all the same equations as in problem 1 (Eq 114) except for $\mathrm{V}_{\mathrm{f}}=224 \mathrm{~L}$ and $\mathrm{V}_{\mathrm{i}}=22.4 \mathrm{~L}, \mathrm{~T}=273 \mathrm{~K}$ Reversible or otherwise, the same equations hold for state functions for isothermal process for ideal gas. |  |
| $\Delta \mathbf{H}=0 \quad$ Answer |  |  |
| $\Delta S=R \ln \left(\mathrm{~V}_{\mathrm{f}} / \mathrm{V}_{\mathrm{i}}\right)$ |  |  |
| $=(1 \mathrm{~mol})\left(8.314 \mathrm{~J} \mathrm{~mol}^{-1} \mathrm{~K}^{-1}\right) \bullet$ |  |  |
| $\ln (224 / 22.4)=19.14 \mathrm{~J} \mathrm{~K}^{-1}$ Answer |  |  |
| $\Delta \mathbf{A}=-(273)\left(19.1 \mathrm{~J} \mathrm{~K}^{-1}\right)$ |  |  |
| =-5226 J Answer |  |  |
| $\Delta \mathbf{G}=-(273)\left(19.1 \mathrm{~J} \mathrm{~K}^{-1}\right)$ |  |  |
| $=-5226 \mathrm{~J}$ Answer |  |  |


| $\delta \mathrm{W}=-\mathrm{p}_{\text {op }} \mathrm{dV}$ | Definition of pV work | 1 |
| :---: | :---: | :---: |
| $\begin{aligned} & p_{\mathrm{op}}=p_{\text {gas }} \\ & \mathrm{p}_{\mathrm{gas}}=(1) \mathrm{RT} / \mathrm{V} \end{aligned}$ | reversible <br> ideal gas equation of state, 1 mole | 2 <br> 3 |
| $\begin{aligned} & \mathrm{\delta W}=-\mathrm{RTdV} / \mathrm{V} \\ & \mathrm{~W}=-\mathrm{JRTdV} / \mathrm{V}=-(1 \mathrm{~mol})(8.314 \mathrm{~J} \\ & \left.\mathrm{mol}^{-1} \mathrm{~K}^{-1}\right) 273 \cdot \ln (224 / 22.4) \\ & =-5226 \mathrm{~J} \quad \text { Answer } \end{aligned}$ | From Eq 1, 2 and 3 Integrating | 4 |
| $\mathrm{d} \mathbf{U}=\delta q+\delta W$ | First Law of Thermodynamics | 5 |
| $\Delta \mathbf{U}=0$ | Already found |  |
| $\begin{aligned} & 0=q+W \\ & q=-W=+5226 J \end{aligned}$ <br> Answer | From Eq 5 <br> This is $q$ for the system | 6 |
| $\begin{aligned} & \Delta \mathbf{S}_{\text {surr }}=-5226 \mathrm{~J} / 273 \mathrm{~K}=-19.14 \mathrm{~J} \\ & \mathrm{~K}^{-1} \\ & \Delta \mathbf{S} \text { universe }=\Delta \mathbf{S}_{\text {gas }}+\Delta \mathbf{S}_{\text {surr }}= \\ & 19.14-19.14=00 \mathrm{~J} \mathrm{~K}^{-1} \text { as expected } \\ & \text { for a reversible process Answer } \end{aligned}$ | q for the surroundings is -5226 J and $\mathrm{T}=273$ | 7 |

(b) ideal gas, $1 \mathrm{~mol} 22.4 \mathrm{~L}, 0^{\circ} \mathrm{C} \rightarrow 224 \mathrm{~L}, 0^{\circ} \mathrm{C} \quad$ irreversible expansion into evacuated vessel

| Equation | Basis for the equation | Eq. \\| |
| :---: | :---: | :---: |
| $\begin{aligned} & \Delta \mathbf{U}=0 \\ & \Delta \mathbf{H}=0 \\ & \Delta \mathbf{S}=19.14 \mathrm{~J} \mathrm{~K}^{-1} \\ & \Delta \mathbf{A}=-5226 \mathrm{~J} \\ & \Delta \mathbf{G}=-5226 \mathrm{~J} \\ & \hline \end{aligned}$ <br> Answer | The same values hold for changes in state functions for identical initial and final states, regardless of the process that brought it about. | 1 |
| $\delta \mathrm{W}=-\mathrm{p}_{\mathrm{op}} \mathrm{dV}$ | Definition of pV work | 2 |
| $\mathrm{p}_{\mathrm{op}}=0$ | Expanding into a vacuum, the gas encounters zero opposing pressure | 3 |
| $\mathrm{W}=0 \quad$ Answer | Since $\mathrm{p}_{\mathrm{op}}=0$ | 4 |
| $\mathrm{d} \mathbf{U}=\delta q+\delta W$ | First law of thermodynamics | 5 |
| $\mathrm{q}=0 \quad$ Answer | From Eq 4 and 5 | 6 |
| $\mathrm{q}_{\text {surroundings }}=0$ | Since no heat passed between gas and its surroundings | 7 |
| $\Delta \mathbf{S}_{\text {surr }}=0$ | Can imagine a reversible process by which the surroundings accepted zero heat from the gas at $0^{\circ} \mathrm{C}$ | 8 |
| $\begin{aligned} & \Delta \mathbf{S} \text { universe }=\Delta \mathbf{S}_{\text {gas }}+\Delta \mathbf{S}_{\text {surr }} \\ & =19.14 \mathrm{~J} \mathrm{~K}^{-1} \end{aligned}$ | $\Delta \mathbf{S}$ universe is positive, consistent with a spontaneous expansion of a gas into vacuum | 9 |

3. ideal gas, $1 \mathrm{~mol} \mathrm{~V}_{1}, \mathrm{~T}_{1} \rightarrow \mathrm{~V}_{2}, \mathrm{~T}_{2}$ irreversible adiabatic expansion no work done

| Equation | Basis for the equation | Eq. <br> \# |
| :---: | :---: | :---: |
| $\mathrm{W}=0$ | Given | 1 |
| $\mathrm{q}=0$ | Given (adiabatic) | 2 |
| $\mathrm{d} \mathbf{U}=\delta q+\delta W$ | First law of thermodynamics | 3 |
| $\Delta \mathbf{U}=0$ | From Eq 1, 2 and 3 | 4 |
| $\mathrm{d} \mathbf{U}=(\partial \mathbf{U} / \partial \mathrm{T})_{\mathrm{V}} \mathrm{dT}+(\partial \mathbf{U} / \partial \mathrm{V})_{T} \mathrm{dV}$ | $\mathbf{U}=\mathbf{U}(\mathrm{T}, \mathrm{V})$ | 5 |
| $\Delta \mathbf{U}=\int \mathrm{C}_{V} \mathrm{dT}$ | For an ideal gas, $(\partial \mathbf{U} / \partial \mathrm{V})_{T}=0$ | 6 |
| $\mathrm{dT}=0 \quad$ Answer | Since $\Delta \mathbf{U}=0$ and $C_{V}$ cannot be zero This is a consequence of the condition that no work is done by the gas. | 7 |
| $\begin{aligned} & \mathrm{d} \mathbf{S}=(\partial \mathbf{S} / \partial \mathrm{T})_{\mathrm{V}} \mathrm{dT}+(\partial \mathbf{S} / \partial \mathrm{V})_{\mathrm{T}} \mathrm{dV} \\ & (\partial \mathbf{S} / \partial \mathrm{V})_{T}=(\partial \mathrm{P} / \partial T)_{V} \\ & (\partial \mathrm{p} / \partial \mathrm{T})_{\mathrm{V}}=\mathrm{R} / \mathrm{V} \\ & (\partial \mathbf{S} / \partial \mathrm{V})_{\mathrm{T}}=\mathrm{R} / \mathrm{V} \\ & \Delta \mathbf{S}=\int(\mathrm{R} / \mathrm{V}) \mathrm{dV} \\ & \hline \end{aligned}$ | Using the same arguments as in the previous problem for 1 mole of an ideal gas undergoing a volume change and also $\mathrm{dT}=0$ | 8 |
| $\Delta \mathbf{S}=\mathrm{R} \ln \left(\mathrm{V}_{2} / \mathrm{V}_{1}\right)$ | This is positive since $V_{2}>V_{1}$ in an expansion |  |
| $\mathrm{q}_{\text {surroundings }}=0$ | Since no heat passed between gas and its surroundings | 7 |
| $\Delta \mathbf{S}_{\text {surr }}=0$ | Can imagine a reversible process by which the surroundings accepted zero heat from the gas at $\mathrm{T}_{1}$ | 8 |
| $\begin{aligned} & \Delta \mathbf{S} \text { universe }=\Delta \mathbf{S}_{\text {gas }}+\Delta \mathbf{S}_{\text {surr }} \\ & =\mathrm{R} \ln \left(\mathrm{~V}_{2} / \mathrm{V}_{1}\right) \end{aligned}$ | $\Delta \mathbf{S}$ universe is positive, consistent with an irreversible process for the gas | 9 |
| $\Delta \mathbf{S}_{\text {gas }}=\mathrm{R} \ln \left(\mathrm{V}_{2} / \mathrm{V}_{1}\right)$ since $\mathrm{dT}=0$, ideal gas $\begin{aligned} & \Delta U=0, q=-W \\ & W=-\int R T d V / V=-R T \ln \left(V_{2} / V_{1}\right) \\ & \mathrm{q}_{\text {gas }}=R T \ln \left(\mathrm{~V}_{2} / \mathrm{V}_{1}\right) \\ & \mathrm{q}_{\text {surroundings }}=-\mathrm{RT} \ln \left(\mathrm{~V}_{2} / \mathrm{V}_{1}\right) \\ & \Delta \mathbf{S}_{\text {surr }}=-\mathrm{R} \ln \left(\mathrm{~V}_{2} / \mathrm{V}_{1}\right) \\ & \Delta \mathbf{S} \text { universe }=\Delta \mathbf{S}_{\text {gas }}+\Delta \mathbf{S}_{\text {surr }}=0 \\ & \text { as expected for a reversible } \\ & \text { process } \end{aligned}$ | If on the other hand, reversible isothermal process for 1 mol ideal gas from $V_{1}$ to $V_{2}$, then conditions are the same as in Problem 2 and $q=-W$ where $W$ can be calculated from $p_{o p}=p_{\text {gas }}$ leading to $q_{\text {surroundings }} / T$ opposite sign to $\Delta \mathbf{S}_{\text {gas }}$ | 10 |

4. ideal gas 1 mol in contact with heat reservoir at $25^{\circ} \mathrm{C} 100 \mathrm{~atm} \rightarrow 1 \mathrm{~atm}$

| Equation | Basis for the equation | Eq. <br> $\#$ |
| :--- | :--- | :--- |
| $\mathrm{~d} \mathbf{S}=(\partial \mathbf{S} / \partial \mathrm{T})_{\mathrm{p}} \mathrm{dT}+(\partial \mathbf{S} / \partial \mathrm{p})_{T} \mathrm{dp}$ | $\mathbf{S}=\mathbf{S}(\mathrm{T}, \mathrm{p})$ | 1 |
| $(\partial \mathbf{S} / \partial \mathrm{p})_{T}=-(\partial \mathrm{V} / \partial T)_{p}$ | Derived $($ see lecture notes Part 4) starting from <br> $\mathrm{d} \boldsymbol{U}=\frac{\partial q_{\text {rev }}-p \mathrm{~d} V=T \mathrm{~d} S}{}-p \mathrm{~d} V$ <br> and use of cross derivatives | 2 |


| $\begin{aligned} & \mathrm{V}=\mathrm{RT} / \mathrm{p} \\ & (\partial \mathrm{~V} / \partial \mathrm{T})_{\mathrm{p}}=\mathrm{R} / \mathrm{p} \\ & (\partial \mathrm{~S} / \partial \mathrm{p})_{T}=-\mathrm{R} / \mathrm{p} \end{aligned}$ | Ideal gas equation of state for one mol Differentiating <br> Substituting Eq 4 into Eq 2 | $\begin{array}{\|l\|} \hline 3 \\ 4 \\ 5 \\ \hline \end{array}$ |
| :---: | :---: | :---: |
| $\Delta \mathbf{S}=-\int(R / p) d p$ | Substituting Eq 5 into Eq 1 and using dT $=0$ for this problem | 6 |
| $\begin{aligned} & \Delta \mathbf{S}_{\text {gas }}=-\left(1 \mathrm{~mol}^{\prime}\right)\left(8.314 \mathrm{~J} \mathrm{~mol}^{-1} \mathrm{~K}^{-1}\right) \bullet \\ & \ln (1 / 100)=+38.3 \mathrm{~J} \mathrm{~mol}^{-1} \mathrm{~K}^{-1} \end{aligned}$ | In all cases, for the same initial and final conditions, regardless of whether work is done or not during the process, because $\mathbf{S}$ is a state function | 7 |
| $\Delta \mathbf{U}=0, \Delta \mathbf{U}=\mathrm{q}+\mathrm{W}$ | Ideal gas and first law of thermodynamics | 8 |
| $\mathrm{q}_{\text {system }}=-\mathrm{W}_{\text {system }}$ | In all cases, for the same initial and final conditions, regardless of whether work is done or not during the process, because $\mathbf{U}$ is a state function and the first law of thermodynamics always holds. |  |
| $q_{\text {surr }}=-q_{\text {system }}=W_{\text {system }}$ | Any heat transferred from the system goes into the surroundings. $q_{\text {surr }}$ is negative (the surroundings provide heat to the system) when the gas does some work |  |


|  | $\mathrm{W}_{\text {system }}$ | $\mathrm{q}_{\text {surr }}$ | $\Delta \mathbf{S}_{\text {surr }}=\mathrm{q}_{\text {surr }} / \mathrm{T}$ | $\Delta \mathbf{S}_{\text {gas }}$ | $\Delta \mathbf{S}_{\text {universe }}$ <br> $=\Delta \mathbf{S}_{\text {surr }}+\Delta \mathbf{S}_{\text {gas }}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| (a) | $-2730 \mathrm{cal}=$ <br> -11422 J | -11422 <br> J | $-11422 / 298=-38.3$ <br> $\mathrm{JK} \mathrm{K}^{-1}$ | $+38.3 \mathrm{~J} \mathrm{~K}^{-1}$ | 0 |
| (b) | $-1000 \mathrm{cal}=$ <br> -4184 J | -4184 J | $-4184 / 298=-14.0$ <br> $\mathrm{JK} \mathrm{K}^{-1}$ | $+38.3 \mathrm{~J} \mathrm{~K}^{-1}$ | $+24.3 \mathrm{~J} \mathrm{~K}^{-1}$ |
| (c) | 0 cal | 0 J | 0 | $+38.3 \mathrm{~J} \mathrm{~K}^{-1}$ | $+38.3 \mathrm{~J} \mathrm{~K}^{-1}$ |

5. Third law entropy of $\mathrm{CH}_{3} \mathrm{NO}_{2}$ (gas, $298.1 \mathrm{~K}, 1 \mathrm{~atm}$ ) can be obtained by adding up the entropies for all the transformations needed for
$\mathrm{CH}_{3} \mathrm{NO}_{2}$ (perfect crystal, $0 \mathrm{~K}, 1 \mathrm{~atm}$ ) $\rightarrow \mathrm{CH}_{3} \mathrm{NO}_{2}$ (gas, $298.1 \mathrm{~K}, 1 \mathrm{~atm}$ )
We will use
$\mathrm{d} \mathbf{S}=(\partial \mathbf{S} / \partial \mathrm{T})_{\mathrm{p}} \mathrm{dT}+(\partial \mathbf{S} / \partial \mathrm{p})_{\mathrm{T}} \mathrm{dp}=\left(\mathrm{C}_{\mathrm{p}} / \mathrm{T}\right) \mathrm{dT}-(\partial \mathrm{V} / \partial \mathrm{T})_{\mathrm{p}} \mathrm{dp}$
$(\partial \mathbf{S} / \partial p)_{T}=-(\partial \mathrm{V} / \partial T)_{p}$
$\mathrm{d} \mathbf{S}=\left(\mathrm{C}_{\mathrm{p}} / \mathrm{T}\right) \mathrm{dT}-(\partial \mathrm{V} / \partial \mathrm{T})_{\mathrm{p}} \mathrm{dp}$
Since $p=1 \mathrm{~atm}, \mathrm{dp}=0, \mathrm{dS}=\left(\mathrm{C}_{\mathrm{p}} / \mathrm{T}\right) \mathrm{dT} ; \Delta \mathbf{S}=\int\left(\mathrm{C}_{\mathrm{p}} / \mathrm{T}\right) \mathrm{dT}$
For a phase transformation where $\mathrm{dT}=0$, we will use $\Delta \mathbf{S}=\mathrm{q}_{\mathrm{REV}, \mathrm{p}} / \mathrm{T}_{\text {trans }}=\Delta \mathbf{H}_{\text {trans }} / T_{\text {trans }}$

Given the table of $C_{p}$ (cal mol-1 $K-1$ ) as function of $T$, calculate $C_{p} / T$ values and plot vs T to find the integral of $\left(\mathrm{C}_{\mathrm{p}} / \mathrm{T}\right) \mathrm{dT}$ between 15 K and 298.1 K


| $\mathrm{CH}_{3} \mathrm{NO}_{2}$ (perfect crystal, $0 \mathrm{~K}, 1$ atm) | $\mathbf{S}=0$ at absolute 0 K for a perfect crystal (Third law of thermodynamics) | 0 |
| :---: | :---: | :---: |
| (crystal, $15 \mathrm{~K}, 1 \mathrm{~atm}$ ) | for $\mathrm{dp}=0 \Delta \mathbf{S}_{1}=\int_{0}^{15}\left(\mathrm{C}_{\mathrm{p}} / \mathrm{T}\right) \mathrm{dT}$ <br> For solids, $\mathrm{C}_{\mathrm{p}}$ and $\mathrm{C}_{\mathrm{v}}$ are nearly the same $C_{p}-C_{V}=-T\left(\frac{\partial V}{\partial T}\right)_{p}^{2}\left(\frac{\partial p}{\partial V}\right)_{T}$ <br> because the two factors are small for solids, especially for low $T$. <br> For perfect solids, $\mathrm{C}_{V}$ behaves asymptotically $\lim _{(T \rightarrow 0 K)} C_{V}=a T^{3}$ <br> $\mathrm{C}_{\mathrm{v}}=464.4\left(\mathrm{~T} / \Theta_{\mathrm{D}}\right)^{3} \mathrm{cal} \mathrm{mol}^{-1} \mathrm{~K}^{-1}=a \mathrm{~T}^{3}$ <br> where $\Theta_{\mathrm{D}}=215 \mathrm{~K}$ for Cu , for example, <br> therefore $a=4.67 \times 10^{-5}$ <br> Debye extrapolation from 0 K to 15 K Debye: <br> $\Delta \mathbf{S}_{1}=(1 / 3) \mathrm{a}(15)^{3}=0.05 \mathrm{cal} \mathrm{mol}^{-1} \mathrm{~K}^{-1}$ <br> This seems rather small. | $\Delta \mathbf{S}_{1}$ |
| (crystal, 244.7 K, 1 atm) | Integration from 15 K to $244.7 \mathrm{~K}, \mathrm{dp}=0\left(\Delta \mathbf{S}_{2}\right)$ Integration over the whole range of temperatures 15K to $298.1 \mathrm{~K}^{2}$ gives $30.52 \mathrm{cal} \mathrm{mol}^{-1} \mathrm{~K}^{-1}$ which includes $\left(\Delta \mathbf{S}_{2}+\Delta \mathbf{S}_{4}\right)$ | $\Delta \mathbf{S}_{2}$ |
| (liquid, 244.7 K, 1 atm) | Phase transition at 244.7 K $\Delta \mathbf{H}=\mathrm{q}_{\mathrm{pREV}}=2319 \mathrm{cal} \mathrm{mol}^{-1}$ $\Delta \mathbf{S}_{3}=2319 \mathrm{cal} \mathrm{mol}^{-1} / 244.7 \mathrm{~K}=9.48 \mathrm{cal} \mathrm{mol}^{-1} \mathrm{~K}^{-1}$ | $\Delta \mathbf{S}_{3}$ |
| (liquid, 298.1, 1 atm) | Integration from 244.7 K to $298.1 \mathrm{~K}, \mathrm{dp}=0\left(\Delta \mathbf{S}_{4}\right)$ This is already included in $\left(\Delta \mathbf{S}_{2}+\Delta \mathbf{S}_{4}\right)$ integration. | $\Delta \mathbf{S}_{4}$ |
| (liquid, 298.1, 3.666 cm Hg ) | Integration from 76.0 cm Hg to 3.666 cm Hg at 298.1 K for liquid, $\mathrm{dT}=0$ <br> For the liquid, assume small temperature coefficient of volume, $(\partial \mathrm{V} / \partial \mathrm{T})_{\mathrm{p}} \sim 0$, leads to $(\partial \mathbf{S} / \partial \mathrm{p})_{\mathrm{T}}=-(\partial \mathrm{V} / \partial \mathrm{T})_{\mathrm{p}}$ | $\Delta \mathbf{S}_{5}$ |


|  | $\begin{aligned} & \sim 0 \\ & \Delta \mathbf{S}_{5}=0 \end{aligned}$ |  |
| :---: | :---: | :---: |
| (vapor, 298.1, 3.666 cm Hg ) | Phase transition at $298.1 \mathrm{~K}, 3.666 \mathrm{~cm} \mathrm{Hg}$ $\Delta \mathbf{S}_{6}=9147 \mathrm{cal} \mathrm{mol}^{-1} / 298.1 \mathrm{~K}=30.68 \mathrm{cal} \mathrm{mol}^{-1} \mathrm{~K}^{-1}$ | $\Delta \mathbf{S}_{6}$ |
| (vapor, 298.1, 1 atm) | Integration from 3.666 cm Hg to 76.0 cm Hg at 298.1 <br> K for gas, $\mathrm{dT}=0$ <br> For the vapor, if ideal: $V=R T / p$ $\begin{aligned} (\partial \mathrm{V} / \partial \mathrm{T})_{\mathrm{p}} & =\mathrm{R} / \mathrm{p} \\ (\partial \mathrm{~S} / \partial \mathrm{p})_{\mathrm{T}} & =-\mathrm{R} / \mathrm{p} \\ \Delta \mathbf{S}_{7}=- & \mathrm{R} \ln (76.0 / 3.666)=-6.03 \mathrm{cal} \mathrm{~mol}^{-1} \mathrm{~K}^{-1} \end{aligned}$ | $\Delta \mathbf{S}_{7}$ |
| $\mathbf{S}_{\text {absolute }}=65.70 \mathrm{cal} \mathrm{~mol}^{-1} \mathrm{~K}^{-1}$ <br> Answer | $\begin{aligned} & \mathbf{S}_{\text {absolute }}= \\ & \mathbf{S}(0 \text { K })+\Delta \mathbf{S}_{1}+\Delta \mathbf{S}_{2}+\Delta \mathbf{S}_{3}+\Delta \mathbf{S}_{4}+\Delta \mathbf{S}_{5}+\Delta \mathbf{S}_{6}+\Delta \mathbf{S}_{7} \end{aligned}$ |  |

6. $\mathrm{N}_{2}$ gas at $1000 \mathrm{~atm}, 1000^{\circ} \mathrm{C}$ fugacity $=$ ?

| Equation | Basis for the equation | Eq. \\| |
| :---: | :---: | :---: |
| $\ln f=\int_{0}^{p} \frac{(Z-1)}{p} \mathrm{~d} p$ | Relation of fugacity to pressure (from lecture notes Part 5) | 1 |
| $\mathrm{Z}=\mathrm{pV} / \mathrm{RT}$ | For a non-ideal gas, Z is not 1 | 2 |
| $\mathrm{p}(\mathrm{V}-\mathrm{b})=$ RT $\mathrm{b}=39.1 \times 10^{-3} \mathrm{~L} \mathrm{~mol}^{-1}$ | Equation of state to be used for this gas | 3 |
| $\mathrm{Z}=\mathrm{V} /(\mathrm{V}-\mathrm{b})$ | From Eq 2 and 3 | 4 |
| Z-1 = b/(V-b) |  | 5 |
| (Z-1)/p = b/RT | From Eq 3 and 5 | 6 |
| $\int(Z-1) d p / p=(b / R T) \int_{0}{ }^{p} d p$ | From Eq 6 | 7 |
| In (f/p) $=(\mathrm{b} / \mathrm{RT}) \mathrm{p}$ | From Eq 1 and 7 | 8 |
| $f=\mathrm{p} \bullet \exp (\mathrm{bp} / \mathrm{RT})$ | Eq 8 and definition of In | 9 |
| $f=1000$ atm•exp[0.0391 $\mathrm{L} \mathrm{mol}^{-1}$ <br> - $1000 \mathrm{~atm} /\left(0.08205 \mathrm{~L} \mathrm{~atm} \mathrm{~mol}^{-1} \mathrm{~K}^{-1}\right.$ <br> $\cdot 1273 \mathrm{~K})$ ] $=1450 \mathrm{~atm} \quad$ Answer | Substituting values into Eq 9 and evaluating |  |

7. From given table of $Z$ vs $p$, calculate the following:

| $p$ | 1.0000 | 4.00000 | 7.00000 | 10.0000 | 40.0 | 70.0 | 100.0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $(Z-1) / p$ | -0.00290 | -0.00301 | -0.00303 | -0.00304 | -0.00316 | -0.00319 | -0.00313 |


| Equation | Basis for the equation | Eq. <br> $\#$ |
| :--- | :--- | :--- |
| $\operatorname{In} f=\quad \int_{0}^{\rho} \frac{(Z-1)}{p} \mathrm{~d} p$ | Relation of fugacity to pressure (from lecture notes <br> Part 5) | 1 |
| $(Z-1) / p=-0.0031-2 \times 10^{-6}(p / 1$ atm $)$ | Fit the values in above table to a function of $p$. <br> Nearly independent of $p$. | 2 |


| $\ln (f / \mathrm{p})=$ Integral $\approx-0.31$ | Integral = area under this curve for $\mathrm{p}=0$ to $\mathrm{p}=100$ <br> atm has a value very close to $100 \bullet(-0.0031)$ | 3 |  |  |  |
| :--- | :--- | :--- | :---: | :---: | :---: |
| $f=\mathrm{p} \exp (-0.31)=0.733 \mathrm{p}$ <br> $f=73.3 \mathrm{~atm}$ at $100 \mathrm{~atm}, 200 \mathrm{~K}$ <br> Answer | From Eq 3 rearrange and evaluate |  |  |  |  |
|  |  |  |  |  |  |

8. 

| Equation | Basis for the equation | Eq. \# |
| :---: | :---: | :---: |
| $\mathrm{d} \mathbf{G}=V \mathrm{~d} p-\mathrm{Sd} T$ | One of the four fundamental equations of thermodynamics | 1 |
| $\mathrm{d} \mathbf{G}=(\partial \mathbf{G} / \partial T)_{p} \mathrm{~d} T+(\partial \mathbf{G} / \partial p)_{T} \mathrm{~d} p$ | $\mathbf{G}=\mathbf{G}(\mathrm{T}, \mathrm{p})$ | 2 |
| $(\partial \boldsymbol{G} / \partial p)_{T}=\mathrm{V}$ | From Eq 1 \& 2 | 3 |
| $\begin{aligned} \mathbf{G}= & R T \\ & \ln (p / 1 a t m)+A^{\prime}+B^{\prime} p \\ & +(1 / 2) C^{\prime} p^{2}+(1 / 3) D^{\prime} p^{3} \end{aligned}$ | Given $\mathbf{G}=\mathbf{G}(\mathrm{T}, \mathrm{p})$ functional form | 4 |
| $\mathrm{V}=\mathrm{RT} / \mathrm{p}+\mathrm{B}^{\prime}+\mathrm{C}^{\prime} \mathrm{p}+\mathrm{D}^{\prime} \mathrm{p}^{2}$ <br> This is the equation of state Answer | Taking $(\partial \mathbf{G} / \partial \mathrm{p})_{T \text { and equating it to } \mathrm{V}}$ | 5 |

9. At $100^{\circ} \mathrm{C}$ and 1 atm , water and steam are at equilibrium (this point is on the liquidvapor equilibrium curve for $\mathrm{H}_{2} \mathrm{O}$ ):
water $\left(100{ }^{\circ} \mathrm{C}\right.$ and 1 atm$) \Leftrightarrow$ steam $\left(100^{\circ} \mathrm{C}\right.$ and 1 atm$)$

| Equation | Basis for the equation | Eq. \# |
| :---: | :---: | :---: |
| $\mathbf{H}=\mathbf{U}+\mathrm{pV}$ | Definition | 1 |
| $\mathrm{d} \mathbf{U}=\delta q+\delta W$ | First law of thermodynamics | 2 |
| $\begin{aligned} & \Delta \mathbf{U}=\mathrm{q}_{\mathrm{p}}-\mathrm{p} \Delta \mathrm{~V} \\ & \Delta \mathbf{H}=\Delta \mathbf{U}+\mathrm{p} \Delta V \end{aligned}$ | At constant opposing pressure p | 3 |
| $\Delta \mathbf{H}=\mathrm{q}_{\mathrm{p}}$ | At constant pressure, from 3 | 4 |
| $\Delta \mathbf{S}=\mathrm{q}_{\text {REv }} / \mathrm{T}$ | Second law of thermodynamics | 5 |
| $\Delta \mathbf{S}=\Delta \mathbf{H} / \mathrm{T}$ | $\mathrm{q}_{\text {REV }}$ is also $\mathrm{q}_{\mathrm{p}}$ at constant pressure ( 1 atm ) in this problem, for the phase change at $100^{\circ} \mathrm{C}$ and 1 atm | 6 |
| $\Delta \mathbf{S}_{\text {vaporizn }}=(1.76-0.31) \mathrm{cal} \mathrm{K}^{-1} \mathrm{~g}^{-1}$ | Given $\left(\mathbf{S}_{\boldsymbol{T}}-\mathbf{S}^{\ominus}{ }_{298}\right)=0.31$ cal K $^{-1} \mathrm{~g}^{-1}$ for water at $100^{\circ} \mathrm{C}$ and 1 atm Given $\left(\mathbf{S}_{\mathrm{T}}-\mathbf{S}^{\ominus}{ }_{298}\right)=1.76$ cal K $^{-1} \mathrm{~g}^{-1}$ for steam at $100^{\circ} \mathrm{C}$ and 1 atm | 7 |
| $=\Delta \mathbf{H}_{\text {vaporizn }} / 373.1 \mathrm{~K}$ | From Eq 6 | 8 |
| (a) $\Delta \mathbf{H}_{\text {vaporizn }}=373.1(1.76-0.31)$ $=541 \mathrm{cal} \mathrm{g}^{-1}$ Answer | Evaluating | 9 |


| $\begin{aligned} & \text { (b) } \Delta \mathbf{H}_{\text {vaporizn }}= \\ & \left\{\left(\mathbf{H}_{T}-\mathbf{H}^{\ominus}{ }_{298}\right)_{\text {steam }}-\left(\mathbf{H}_{T}-\mathbf{H}^{\ominus}{ }_{298}\right)_{\text {water }}\right\} \\ & \text { at } 100{ }^{\circ} \mathrm{C} \text { and } 1 \mathrm{~atm} \\ & 541 \mathrm{cal} \mathrm{~g}^{-1} \\ & =640 \mathrm{cal} \mathrm{~g}^{-1}-\left(\mathbf{H}_{T}-\mathbf{H}^{\ominus} 298\right)_{\text {water }} \\ & \left(\mathbf{H}_{T}-\mathbf{H}^{\ominus} 298\right)_{\text {water }}=99 \mathrm{cal} \mathrm{~g}^{-1} \end{aligned}$ | Using Eq 9 and given | 10 |
| :---: | :---: | :---: |
| (c) $\mathbf{G}=\mathbf{H}-\mathrm{TS}$ | Definition | 11 |
| $\Delta \mathbf{G}=\Delta \mathbf{H}-\mathrm{T} \Delta \mathbf{S}$ | At constant T | 12 |
| $\begin{aligned} & \left(\mathbf{G}_{T}-\mathbf{G}^{\ominus}{ }_{298}\right)_{\text {water }}=\left(\mathbf{H}_{T}-\mathbf{H}^{\ominus}{ }_{298}\right)_{\text {water }} \\ & -373.1 \bullet\left(\mathbf{S}_{T}-\mathbf{S}^{\ominus} 298\right)_{\text {water }} \\ & =99-373.1 \bullet 0.31=-16.7 \text { cal g }{ }^{-1} \\ & \text { Answer } \end{aligned}$ | Substituting values | 13 |
| $\begin{aligned} & \left(\mathbf{G}_{T}-\mathbf{G}^{\ominus}{ }_{298}\right)_{\text {steam }}=\left(\mathbf{H}_{T}-\mathbf{H}^{\ominus}{ }_{298}\right)_{\text {steam }} \\ & -373.1 \bullet\left(\mathbf{S}_{\mathrm{T}}-\mathbf{S}^{\ominus} 298\right)_{\text {steam }} \\ & =640-373.1 \bullet 1.76=-16.7 \text { cal g }^{-1} \\ & \text { Answer } \end{aligned}$ | The two are equal as expected |  |

10. C (graphite) $\rightarrow \mathrm{C}$ (diamond) (298 K, 1 atm)

| Equation | Basis for the equation | Eq. \# |
| :---: | :---: | :---: |
| $\begin{aligned} & \mathrm{C}_{\text {(diamond) }}+\mathrm{O}_{2} \rightarrow \mathrm{CO}_{2}(298 \mathrm{~K}, 1 \mathrm{~atm}) \\ & \Delta \mathrm{H}=94.484 \mathrm{kcal} \mathrm{~mol}^{-1} \\ & \mathrm{C}_{\text {(graphite) }}+\mathrm{O}_{2} \rightarrow \mathrm{CO}_{2}(298 \mathrm{~K}, 1 \mathrm{~atm}) \\ & \Delta \mathbf{H}=94.030 \mathrm{kcal} \mathrm{~mol}^{-1} \\ & \mathrm{C}_{\text {(graphite) }} \rightarrow \mathrm{C}_{\text {(diamond) }}(298 \mathrm{~K}, 1 \mathrm{~atm}) \\ & \Delta \mathrm{H}=94.484-94.030=0.454 \\ & \mathrm{kcal} \mathrm{~mol} \\ & \text { kil } \end{aligned}$ | H is a state function | 1 |
| $\begin{aligned} & \Delta \mathbf{S}=\mathbf{S}_{\text {diamond }}-\mathbf{S}_{\text {graphite }} \\ & =0.5829-1.3609 \\ & =-0.7780 \mathrm{cal} \mathrm{~K}^{-1} \mathrm{~mol}^{-1} \end{aligned}$ | Use given data at $298 \mathrm{~K}, 1 \mathrm{~atm}$ | 2 |
| $\begin{aligned} & \mathbf{G}=\mathbf{H}-\mathbf{T S} \\ & \Delta \mathbf{G}=\Delta \mathbf{H}-\mathrm{T} \Delta \mathbf{S} \end{aligned}$ | Definition At constant T | $\begin{array}{\|l\|} \hline 3 \\ 4 \end{array}$ |
| $\begin{gathered} \Delta \mathbf{G}=454-298 \bullet(-0.7780) \\ =685.8 \mathrm{cal} \mathrm{~mol}^{-1}(298 \mathrm{~K}, 1 \mathrm{~atm}) \\ \text { Answer } \end{gathered}$ |  | 5 |
| $\mathbf{G}^{\ominus}{ }_{298}(\text { diamond })-\mathbf{G}^{\ominus}{ }_{298} \text { (graphite) }=$ $685.8 \mathrm{cal} \mathrm{~mol}^{-1}$ | ${ }^{\ominus}$ means 1 atm | 6 |
| $\mathrm{d} \boldsymbol{G}=V \mathrm{~d} p-\mathbf{S d} T$ | One of the four fundamental equations of thermodynamics | 7 |
| $\mathbf{G}(T)=\boldsymbol{G}_{T}{ }_{T}+\int_{1}^{p} V d p$ | For any pure material at a given temperature ( $\mathrm{dT}=0$ ) | 8 |


| $\begin{array}{r} \mathbf{G}_{298}(\text { diamond })=\mathbf{G}^{\ominus}{ }_{298}(\text { diamond })+ \\ \int_{1}^{\mathrm{p}} \mathrm{~V}_{\text {diamond }} \mathrm{dp} \\ \mathbf{G}_{298}(\text { graphite })=\mathbf{G}^{\ominus}{ }_{298}(\text { graphite })+ \\ \int_{1}^{\mathrm{p}} \mathrm{~V}_{\text {graphite }} \mathrm{dp} \end{array}$ | Applying Eq 7 to each pure material | 9 |
| :---: | :---: | :---: |
| $\mathrm{C}_{\text {(graphite) }} \Leftrightarrow \mathrm{C}_{\text {(diamond) }}(298 \mathrm{~K}, \mathrm{p} \mathrm{atm})$ <br> $\mathbf{G}_{298}$ (diamond, $\mathbf{p}$ ) $\mathbf{G}_{298}$ (graphite, $\left.\mathbf{p}\right)=0$ | At equilibrium at this value of $p$ means $\mathbf{G}$ of graphite and $\mathbf{G}$ of diamond are equal at this $p$ and 298 K . | 10 |
| $\begin{aligned} 0 & =685.8 \text { cal mol }^{-1}+\int_{1}{ }^{p} V_{\text {diamond }} \mathrm{dp} \\ & -\int_{1} \mathrm{p}_{\text {graphite }} \mathrm{dp} \end{aligned}$ | Using Eq 6, 9 and 10 | 11 |
| $\begin{aligned} & 0=685.8 \mathrm{cal} \mathrm{~mol} \\ &+(1.987 / 0.08206) \bullet \\ &\left(12 \mathrm{~g} / 3.513 \times 10^{3} \mathrm{~g} \mathrm{~L}^{-1}\right)(\mathrm{p}-1) \\ &-\left(12 \mathrm{~g} / 2.260 \times 10^{3} \mathrm{~g} \mathrm{~L}^{-1}\right)(\mathrm{p}-1) \end{aligned}$ | Using given densities, independent of pressure and $1 \mathrm{~mol} \mathrm{C}=12 \mathrm{~g}$ and $10^{3} \mathrm{~cm}^{3}=1 \mathrm{~L}$ and use conversion $1.98722 \mathrm{cal}=0.0820578 \mathrm{~L}$ atm | 12 |
| $\begin{aligned} & 0==685.8-0.04586(\mathrm{p}-1) \\ & \mathrm{p}-1=14950 \text { atm } \quad \text { Answer for } \\ & C_{\text {(graphite) }} \Leftrightarrow C_{\text {(diamond) }}(298 \mathrm{~K}, \mathrm{p} \mathrm{~atm}) \end{aligned}$ | Solving for $p$ |  |
| $\begin{aligned} & \hline \mathrm{C}_{\text {(graphite) }} \Leftrightarrow \mathrm{C}_{\text {(diamond) }}(1273 \mathrm{~K}, \mathrm{p} \text { atm }) \\ & \left.\mathbf{G}_{1273} \text { (diamond, } \mathrm{p}\right)-\mathbf{G}_{1273}(\text { graphite }, \mathrm{p}) \\ & =0 \\ & 0=\mathbf{G}^{\ominus}{ }_{1273}(\text { diamond })- \\ & \mathbf{G}^{\ominus}{ }_{1273}(\text { graphite })+\int_{1}^{\mathrm{p}} \mathrm{~V}_{\text {diamond }} \mathrm{dp} \\ & \quad-\int_{1}^{\mathrm{p}} \mathrm{~V}_{\text {graphite }} \mathrm{dp} \end{aligned}$ <br> Solve for $p$ | To do the same calculation for $1000^{\circ} \mathrm{C}$. <br> We need the value <br> $\mathbf{G}^{\ominus}{ }_{1273}$ (diamond) - $\mathbf{G}^{\ominus}{ }_{1273}$ (graphite), for which we need molar entropies $\mathbf{S}$ at $1000^{\circ} \mathrm{C}$, 1 atm , and also $\Delta \mathrm{H}$ at $1000^{\circ} \mathrm{C}, 1 \mathrm{~atm}$. <br> $\mathrm{S}_{1273}$ for graphite $=1.3609 \mathrm{cal} \mathrm{K}^{-1} \mathrm{~mol}^{-1}+$ $\int_{298}{ }^{1273}\left(\mathrm{C}_{\mathrm{p}, \text { graphite }} / \mathrm{T}\right) \mathrm{dT}$ <br> $\mathbf{S}_{1273}$ for diamond $=0.5829 \mathrm{cal} \mathrm{K}^{-1} \mathrm{~mol}^{-1}+$ $\int_{298}{ }^{1273}\left(\mathrm{C}_{\mathrm{p} \text {, diamond }} / \mathrm{T}\right) \mathrm{dT}$ <br> $\Delta \mathbf{H}_{1273}$ (graphite) $=\Delta \mathbf{H}_{298}+\int_{298}{ }^{1273} C_{p, \text { graphite }} d T$ $\Delta \mathbf{H}_{1273}$ (diamond) $=\Delta \mathbf{H}_{298}+\int_{298}{ }^{1273} \mathrm{C}_{\mathrm{p}, \text { diamond }} \mathrm{dT}$ To estimate these quantities, we need to find $\mathrm{C}_{\mathrm{p}}$ data for the two forms over the range 298 to 1273K. |  |

