

Name _____

Please put answers within boxes provided.

Chemistry 342

November 23, 1998

Third Exam

Answers

1. In the following figures (a)-(h) attach these equation labels (A)-(Z') to each one of the appropriate straight or curved lines to which the equation applies. An equation may be used more than once, some equations may not be used at all, and some lines may not have applicable equations from the list.

Equations:

(A) $p_A/p_A^* = x_A$

(B) $y_A = p_A^*/p$

(C) $y_A - X_A = (n_{A(VAP)} / n_{(VAP)}) - (n_A / n)$

(D) $p = p_A^* p_B^* / \{ p_A^* + (p_B^* - p_A^*) y_A \}$

(E) $p = p_A + p_B$

(F) $p_B = K_H x_B$

(G) $p = p_A^* x_A + p_B^* (1 - x_A)$

(H) $dp/dT = \Delta_{vap} H / \{ T(V_{m,gas} - V_{m,liquid}) \}$

(I) $\ln x_A = \Delta_{fus} H / R \left[\frac{-1}{T} + \frac{1}{T^*} \right]$

(J) $(\partial \mu_{gas} / \partial T)_p = -S_{m,gas}$

(K) $\mu_{A,vap}^* = \mu_T^\ominus + RT \ln(p_A^*/1)$

(L) $\mu_{A,liquid}(T, p, x_A) = \mu_{A,solid}(T, p)$

(M) $\mu_A(T) = \mu_A^*(T) + RT \ln x_A$

(N) $(\partial [\Delta_{rxn} G^\ominus / T] / \partial T)_p = -(\Delta_{rxn} H^\ominus / T^2)$

(O) $\ln \gamma_\pm = z_+ z_- I^{1/2} (e^2 / 10 \epsilon k_B T)^{3/2} \{ 2 \pi \rho N_{Avog} \}^{1/2}$

(P) $g_B(T, p) = \mu_B^*(T, p) + RT \ln K_H / p_B^*$

(Q) $\ln K = 2 \ln C_s + 2 \ln \gamma_\pm$

(R) $\mathcal{E} = \mathcal{E}^\ominus - (RT/v \mathcal{F}) \ln Q_{rxn}$

(S) $F = 0$

(T) $F = 1 \quad F' = 0$

(U) $F = 2 \quad F' = 1$

(V) $\mu(T) = \mu_T^\ominus + RT \ln f$

(W) $\ln f = \int_0^p \frac{(Z-1)}{p} dp$

(X) $\mathcal{E} dQ = \mu'_e dn - \mu_e dn$

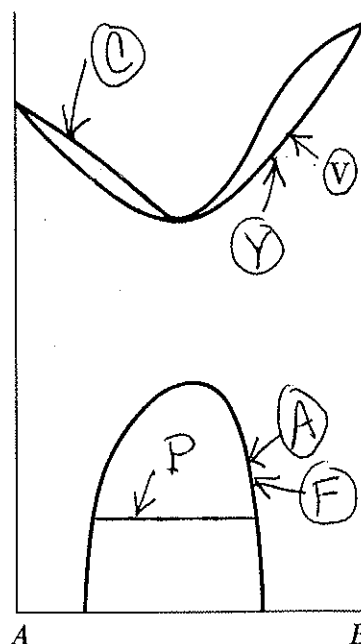
(X') $G_{T,p} = \sum_i n_i \mu_i$

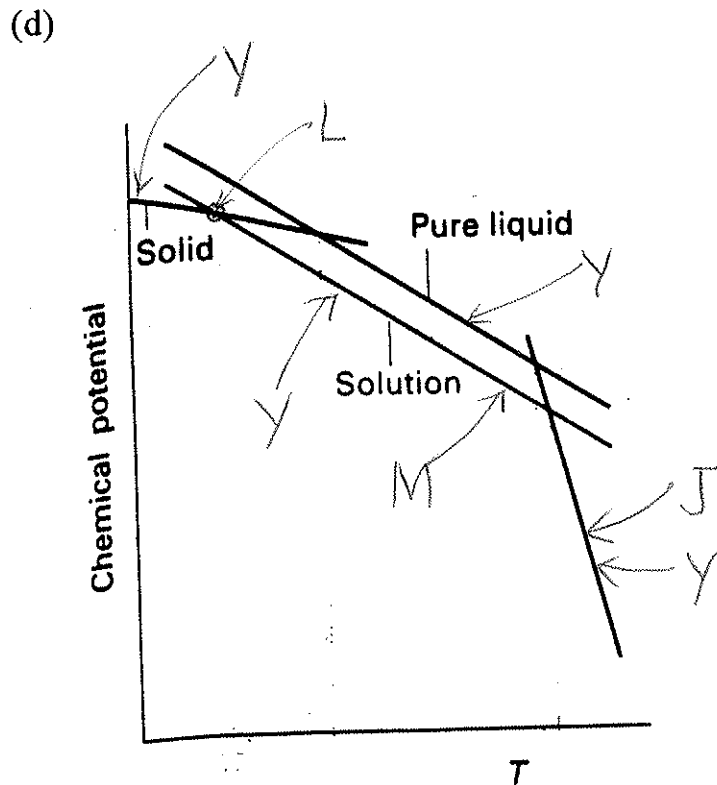
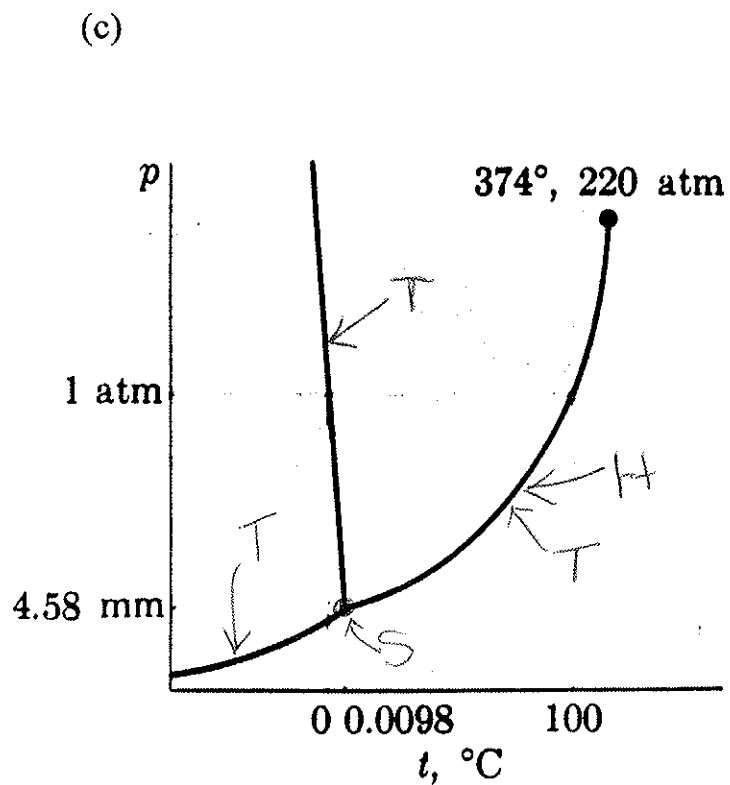
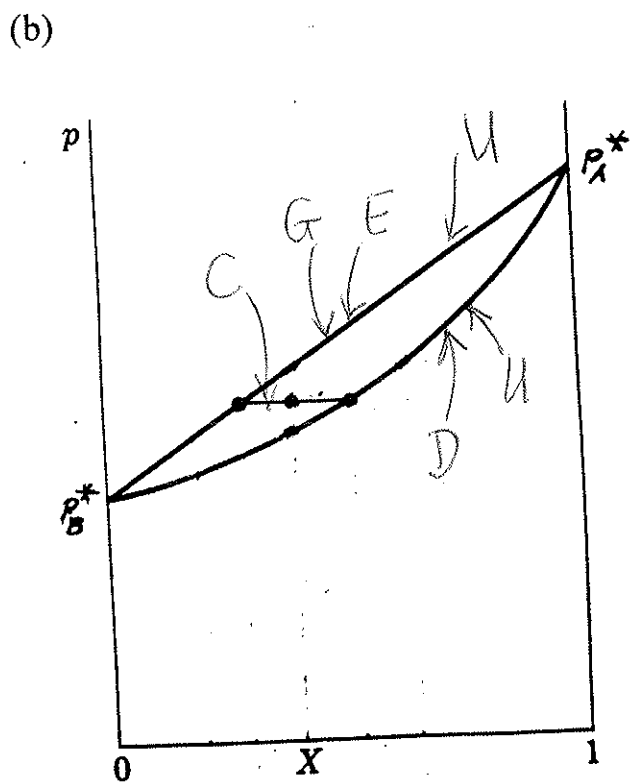
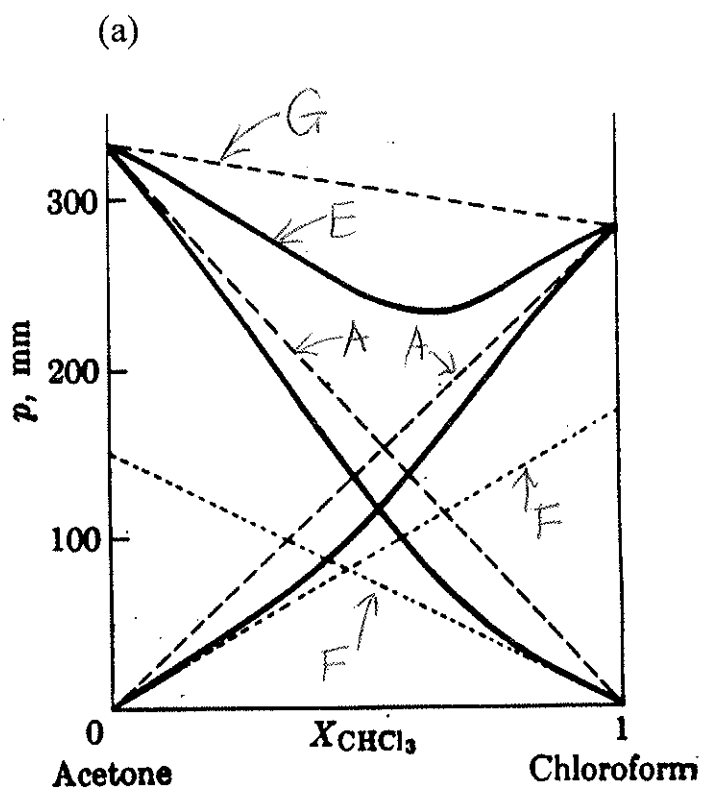
(Y) $(\partial G / \partial T)_p = -S$

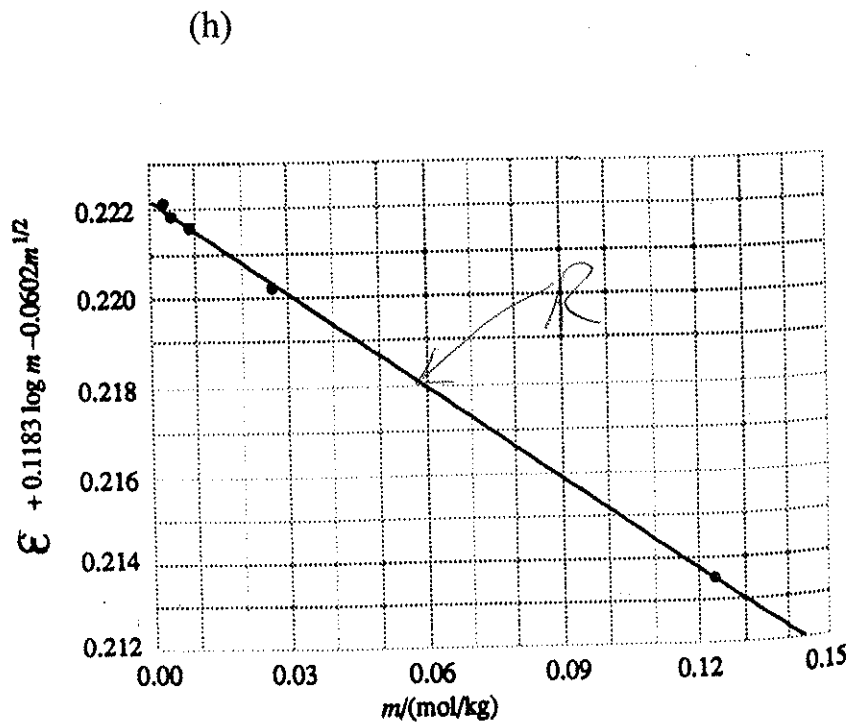
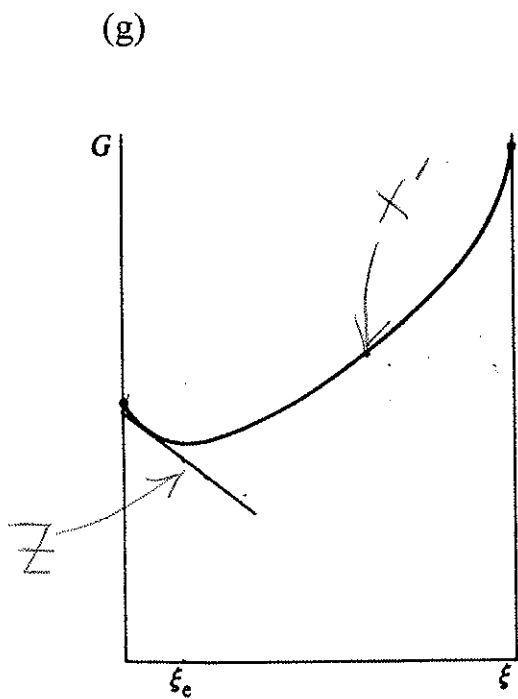
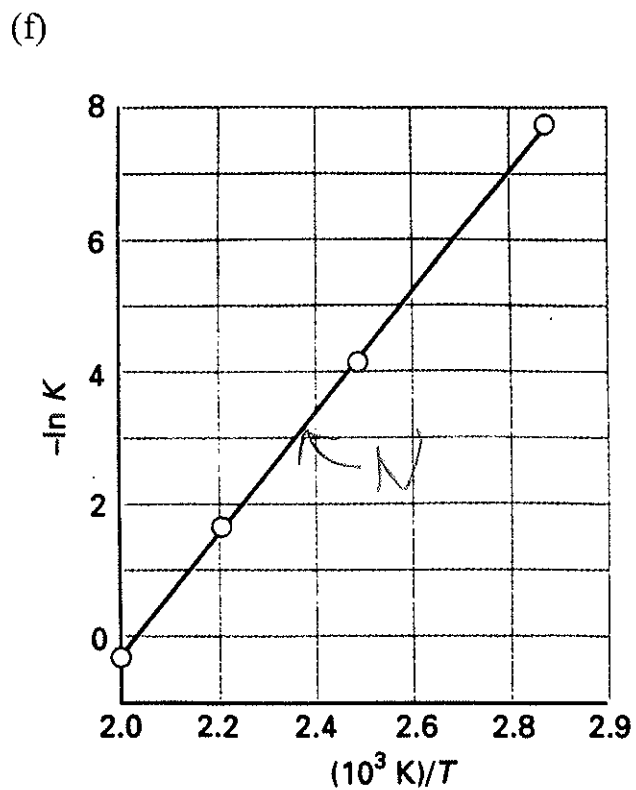
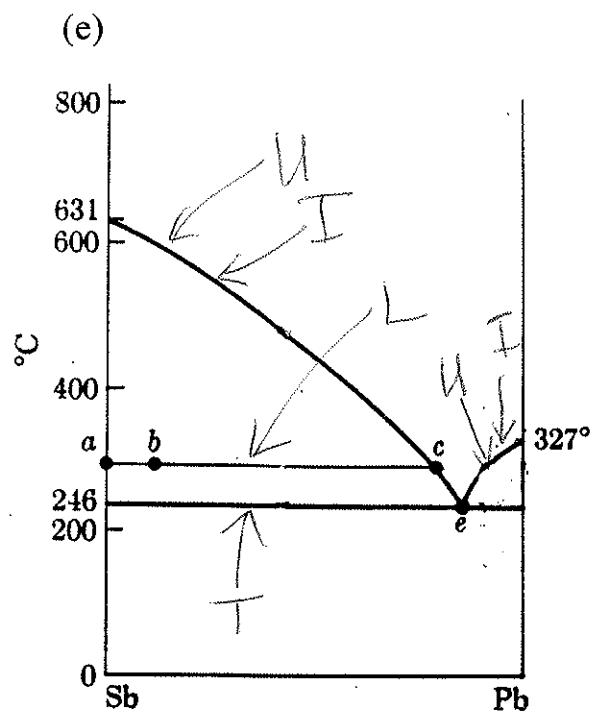
(Z) $\Delta_{rxn} G = (c\mu_C + e\mu_E) - (a\mu_A + b\mu_B)$

(Z') $\Delta_{rxn} H^\ominus = -v \mathcal{F} \{ \mathcal{E}^\ominus - T(d\mathcal{E}^\ominus/dT) \}$

Your answer should look like the following example:







$$1 \text{ J} = 1 \text{ kg m}^2 \text{ s}^{-2} \quad k_B = 1.38066 \times 10^{-23} \text{ J K}^{-1}$$

$$R = N_{\text{Avogadro}} k_B$$

$$R = 8.31441 \text{ J mol}^{-1} \text{ K}^{-1} = 1.98718 \text{ cal mol}^{-1} \text{ K}^{-1} = 0.082057 \text{ L atm mol}^{-1} \text{ K}^{-1}$$

The limiting Debye-Hückel eqn: $\ln \gamma_{\pm} = z_+ z_- I^{1/2} (e^2 / 10 \epsilon k_B T)^{3/2} \{2 \pi \rho N_{\text{Avog}}\}^{1/2}$

for water at 25°C as the solvent, $\ln \gamma_{\pm} = z_+ z_- I^{1/2} (1.17223)$

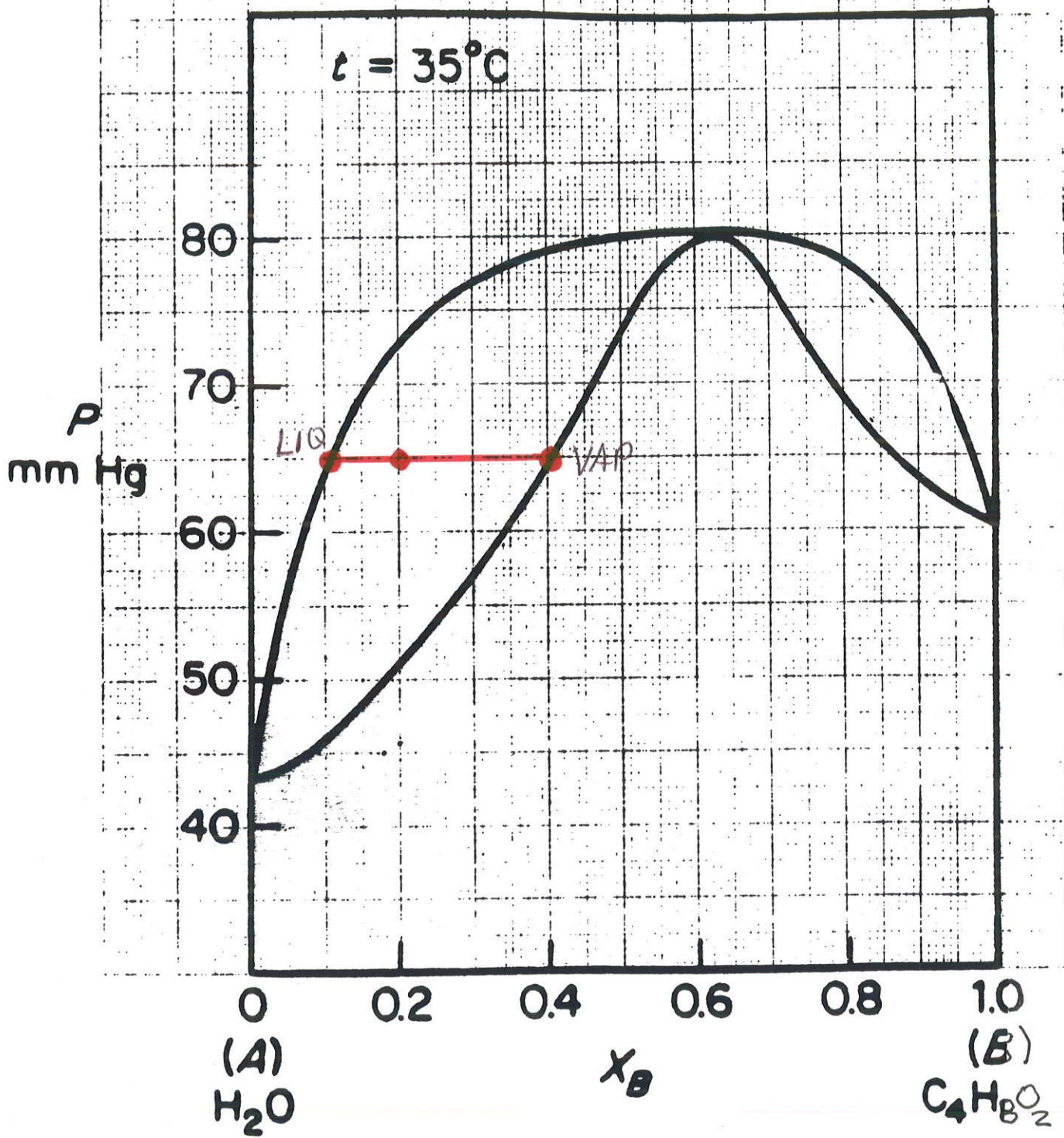
$$\text{ionic strength} = \frac{1}{2} \sum m_i z_i^2$$

$$\ln x_A = \Delta_{\text{fus}} H / R \left[\frac{-1}{T} + \frac{1}{T^*} \right]$$

2. From the attached graph determine the following and place your answers in the table below:

- The mole fraction of B in the vapor at 35°C and a pressure of 65 mm Hg for a system having over all 20 mole % B
- The mole fraction of A in the liquid at 35°C and a pressure of 65 mm Hg for a system having over all 20 mole % B
- The relative amounts of liquid and vapor for a system having over all 20 mole % B at 35°C and a pressure of 65 mm Hg
- Are the A-B intermolecular interactions stronger or weaker (which one?) than the A-A and B-B interactions?
- What is the vapor pressure of pure A at 35°C?
- What is the vapor pressure of pure B at 35°C?
- If an azeotrope is formed by this system, what is the approximate composition of the azeotropic mixture?
- If an azeotrope is formed by this system, would it boil at a higher or lower temperature (which one?) than both A and B?
- Would the activity coefficient of B in a liquid solution of A+B be smaller or greater (which one?) than 1.0?
- Would the activity coefficient of A in a liquid solution of A+B be smaller or greater (which one?) than 1.0?

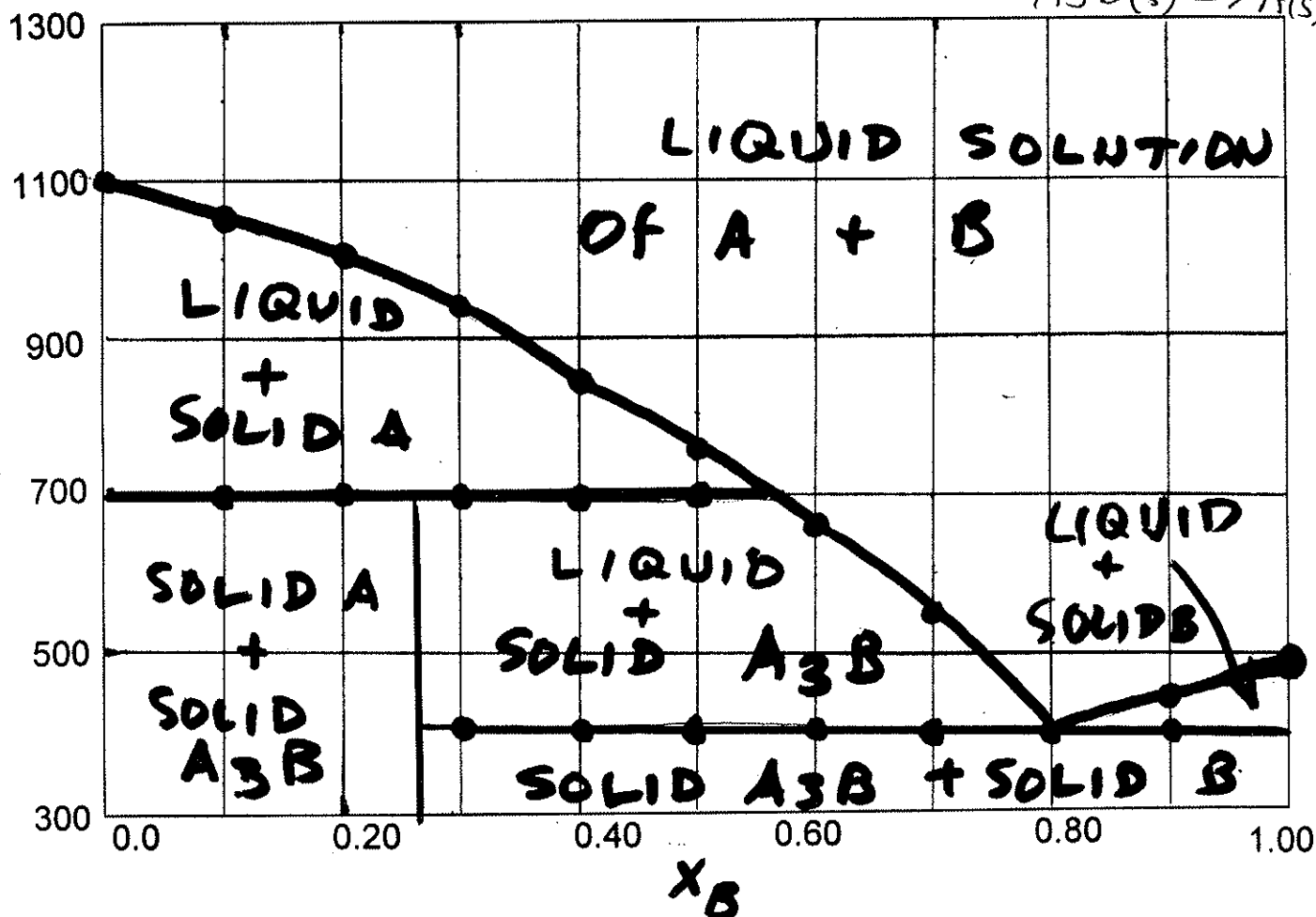
| (a) | (b) | (c) | (d) | (e) |
|--------------|--------------|---|---------|---------|
| $x_B = 0.40$ | $x_A = 0.90$ | $\frac{n_{\text{LIQ}}}{n_{\text{VAP}}} \approx 2$ | weaker | 43 mm |
| (f) | (g) | (h) | (i) | (j) |
| 60 mm | $x_B = 0.6$ | LOWER | greater | greater |



3. The table below gives the break and halt (plateau in the cooling curve) temperatures found in the cooling curves of two metals A and B. Construct a phase diagram consistent with the data of these curves. Label the regions of the diagram, stating what phases and substances are present. Give the probable formula of any compounds that form

| X_B | break, t °C | first halt, t °C | second halt, t °C |
|-------|---------------|--------------------|---------------------|
| 0.0 | | 1100 | |
| 0.10 | 1060 | 700 | |
| 0.20 | 1000 | 700 | |
| 0.30 | 940 | 700 | 400 |
| 0.40 | 850 | 700 | 400 |
| 0.50 | 750 | 700 | 400 |
| 0.60 | 670 | 400 | |
| 0.70 | 550 | 400 | |
| 0.80 | | 400 | |
| 0.90 | 450 | 400 | |
| 1.00 | | 500 | |

Assume simplest formula for compound A_3B which melts incongruently at 700°C undergoing the peritectic reaction $A_3B(s) \rightarrow A(s) + \text{LIQUID}$



4. For the reaction: $\text{C}(\text{graphite}) + 2\text{H}_2(\text{g}) \rightarrow \text{CH}_4(\text{g})$

the change in enthalpy is $-21\,045 \text{ cal mol}^{-1}$ at 600°C .

The third law entropies ($\text{cal mol}^{-1} \text{K}^{-1}$ at 600°C and 1 atm are:

4.98 for $\text{C}(\text{graphite})$; 38.9 for $\text{H}_2(\text{g})$; 56.6 for $\text{CH}_4(\text{g})$

(a) Calculate the equilibrium constant for the reaction at 600°C .

$$\Delta_{\text{rxn}} G^\ominus_T = -RT \ln K_p$$

$$\Delta_{\text{rxn}} G^\ominus_T = \Delta_{\text{rxn}} H^\ominus_T - T \Delta_{\text{rxn}} S^\ominus_T$$

$$\Delta_{\text{rxn}} S^\ominus_T = S^\ominus_{\text{CH}_4, T} - 2S^\ominus_{\text{H}_2, T} - S^\ominus_{\text{graphite}, T}$$

$$\Delta_{\text{rxn}} S^\ominus_{873} = 56.6 - 2(38.9) - 4.98 = -26.18 \text{ cal mol}^{-1} \text{K}^{-1}$$

$$\Delta_{\text{rxn}} H^\ominus_{873} = -21045 \text{ cal mol}^{-1}$$

$$\Delta_{\text{rxn}} G^\ominus_{873} = -21045 - (873)(-26.18) = 1810 \text{ cal mol}^{-1} = -RT \ln K_p$$

$$-\ln K_p = 1810 / (1.987)(873) = 1.0435$$

$$K_p = 0.352$$

(b) What experimental information would you need, besides that given above, in order to be able to evaluate K_p at any temperature? Make a suitable approximation that is adequate over a short temperature range and use it to estimate the equilibrium constant at 750°C .

Consider G/T :

$$(\partial(G/T) / \partial T)_p = -(G/T^2) + (1/T)(\partial G / \partial T)_p$$

using $G \equiv H - TS$

$$(\partial(G/T) / \partial T)_p = -(H/T^2) + (S/T) + (1/T)(\partial G / \partial T)_p$$

$$\text{Recall that: } (\partial G / \partial T)_p = -S$$

$$\text{Thus, } (\partial(G/T) / \partial T)_p = -(H/T^2)$$

Combine equations like this for each of reactants and products:

$$(\partial(\Delta_{\text{rxn}} G^\ominus_T / T) / \partial T)_p = -(\Delta_{\text{rxn}} H^\ominus_T / T^2)$$

$$\Delta_{\text{rxn}} G^\ominus_T = -RT \ln K_p$$

$$\therefore (\partial \ln K_p / \partial T)_p = (\Delta_{\text{rxn}} H^\ominus_T / RT^2) \text{ Integrating we get}$$

$$\int d \ln K_p = \int (\Delta_{\text{rxn}} H^\ominus_T / R T^2) dT \text{ We need to know } T \text{ dependence of } \Delta_{\text{rxn}} H^\ominus_T.$$

(for that we need heat capacities of reactants and products). If we assume that the change with T is small, then approximately,

$$\int d \ln K_p = +(\Delta_{\text{rxn}} H^\ominus_T / R) \int T^{-2} dT \quad \text{the integral } \int T^{-2} dT \text{ is } -\{(1/T_2) - (1/T_1)\}$$

$$\ln(K_{p, 1023} / K_{p, 873}) = (\Delta_{\text{rxn}} H^\ominus_T / R) \{ (1/873) - (1/1023) \}$$

$$= (-21045 \text{ cal mol}^{-1} / 1.987) \{ (1/873) - (1/1023) \} = -1.779$$

$$(K_{p, 1023} / K_{p, 873}) = 0.169 \quad K_{p, 1023} = 0.0594$$

(c) Is it better to use high or low temperatures to get the maximum yield of $\text{CH}_4(\text{g})$? High or low pressures? Explain.

| | |
|---|---|
| T low T since K is found to be larger at lower T | p high pressure favors producing fewer gas molecules: 1CH_4 for every 2H_2 reacted |
|---|---|

5 The activity of pure liquid water at 1 atm is 1 according to the definition of the standard state as a pure liquid at 1 atm. Calculate the activity of pure liquid water at 50°C and 10^4 atm, given that the integral $\int V_m dp$ between 1 atm and 10^4 atm is $= 161.450 \text{ L atm}$ for liquid water at 50°C .

$$dG = -SdT + Vdp$$

$$d\mu_{\text{LIQ}}(T,p) = Vdp$$

$$\mu_{\text{LIQ}}(T,p_2) - \mu_{\text{LIQ}}(T,1 \text{ atm}) = \int V_m dp$$

Definition of activity:

$$\mu_{\text{LIQ}}(T,p_2) = \mu_{\text{LIQ}}^\ominus_T + RT \ln a_{\text{LIQ}}(T,p_2)$$

$$\mu_{\text{LIQ}}(T,p_2) - \mu_{\text{LIQ}}(T,1 \text{ atm}) = RT \ln a_{\text{LIQ}}(T,p_2)/a_{\text{LIQ}}(T,1 \text{ atm})$$

$$a_{\text{LIQ}}(T,1 \text{ atm}) = 1$$

$$\text{Therefore, } \int V_m dp = RT \ln a_{\text{LIQ}}(T,p_2)/1$$

$$161.450 \text{ L atm} = RT \ln a_{\text{LIQ}}(323 \text{ K}, 10^4 \text{ atm})$$

$$\ln a_{\text{LIQ}}(323 \text{ K}, 10^4 \text{ atm}) = 161.450/(0.0820567 \times 323) = 6.09$$

$$a_{\text{LIQ}}(323 \text{ K}, 10^4 \text{ atm}) = 439$$

ASIDE: Incidentally, $a_{\text{LIQ}}(T,p_2)$ is also $= f_{\text{GAS}}(T,p_2)/f_{\text{GAS}}(T,1 \text{ atm})$ which reinforces the identification of activity with the chemical potential. This is how we do it for this system:

$$\text{Liquid and vapor at equilibrium } \mu_{\text{LIQ}}(T,1 \text{ atm}) = \mu_{\text{VAP}}(T,1 \text{ atm}) \quad (2)$$

$$\mu_{\text{LIQ}}(T,p_2) = \mu_{\text{VAP}}(T,p_2) \quad (1)$$

subtract equation (2) from eq(1)

$$\mu_{\text{LIQ}}(T,p_2) - \mu_{\text{LIQ}}(T,1 \text{ atm}) = \mu_{\text{VAP}}(T,p_2) - \mu_{\text{VAP}}(T,1 \text{ atm})$$

But,

$$\mu_{\text{VAP}}(T,p_2) = \mu_{\text{GAS}}^\ominus_T + RT \ln f(T,p_2) \quad (3)$$

$$\mu_{\text{VAP}}(T,1 \text{ atm}) = \mu_{\text{GAS}}^\ominus_T + RT \ln f(T,1 \text{ atm}) \quad (4)$$

$$\text{Therefore, } \mu_{\text{LIQ}}(T,p_2) - \mu_{\text{LIQ}}(T,1 \text{ atm}) = RT \ln f_{\text{GAS}}(T,p_2)/f_{\text{GAS}}(T,1 \text{ atm})$$