

Problem Set 2

On Eigenvalues and Eigenfunctions

Some useful identities:

$$i \equiv \sqrt{-1} \quad \exp(i x) \equiv \cos x + i \sin x \quad \exp(-i x) \equiv \cos x - i \sin x$$

1. (a) Show that $\exp[ax]$ is an eigenfunction of the operator d/dx , and find the corresponding eigenvalue.
(b) Which of the following functions are eigenfunctions of the operator d/dx and which of d^2/dx^2 ? Give the eigenvalues where appropriate.
 - (i) $\exp(ikx)$
 - (ii) $\cos(kx)$
 - (iii) $\exp[-ax^2]$

2. An eigenvalue equation that we will encounter in this course has the form $d^2/dx^2 \Psi(x) = -C\Psi(x)$, where C is a positive constant. Find the general form for the eigenfunction $\Psi(x)$. To the extent possible, evaluate the constants that appear in the equation in terms of the constant C .

HINT: The only two common functions whose second derivative is a constant times the function itself are exponentials and trigonometric sine or cosine functions.

Thus, investigate two possibilities for the general form for the eigenfunction $\Psi(x)$:

Possibility 1: $\Psi_1(x) = A \sin(bx) + B \cos(bx)$

Possibility 2: $\Psi_2(x) = A \exp(bx) + B \exp(-bx)$

In both possibilities b is real.

3. Another eigenvalue equation that we will encounter in this course has the form $d^2/dx^2 \Psi(x) = +C\Psi(x)$, where C is a positive constant. Find the general form for the eigenfunction $\Psi(x)$. To the extent possible, evaluate the constants that appear in the equation in terms of the constant C . See above problem for the hint.