Problem Set 2 Answers
On Eigenvalues and Eigenfunctions

1. (a) Show that exp[ax] is an eigenfunction of the operator d/dx, and find the corresponding eigenvalue.

d/dx exp[ax] = a exp[ax] is of the form d/dx $\Psi(x) = a\Psi(x)$ therefore exp[ax] is an eigenfunction of the operator d/dx, with eigenvalue a.

(b) Which of the following functions are eigenfunctions of the operator d/dx and which of d^2/dx^2? Give the eigenvalues where appropriate.

function $\psi(x)$	d/dx $\psi(x)$	eigenvalue?	$d^2/dx^2\ \psi(x)$	eigenvalue ?
$\exp(ikx)$	$ik\exp(ikx)$	ik	$-k^2\exp(ikx)$	$-k^2$
cos(kx)	- ksin(kx)	no	$-k^2\cos(kx)$	$-k^2$
$\exp[-ax^2]$	$-a2x\exp[-ax^2]$	no	$\{4a^2x^2-2a\}\exp[-ax^2]$	no

2. (a) $d^2/dx^2\Psi(x) = -C\Psi(x)$ where C is a <u>positive</u> constant

d/dx sinx = cos x ; d/dx cosx = -sin x

Possible solutions	d/dx Ψ (x)	$d^2/dx^2\Psi$ (x)	rearrange	eigenvalue? -C =
$\Psi_1(x)$ = A sin(bx) + Bcos(bx) where b is real	Abcos(bx) -Bbsin(bx)	$-Ab^2\sin(bx)$ $-Bb^2\cos(bx)$	$-b^2\ \Psi_1(x)$	$-b^2$ thus $b=\pm\sqrt{C}$
$\Psi_2(x)$ = Aexp(bx) + Bexp(-bx) where b is real	Ab exp(bx) -Bb exp(-bx)	$Ab^2\exp(bx)$ $+Bb^2\exp(-bx)$	$b^2\Psi_1(x)$	b^2 thus $b=\pm\sqrt{-C}$ $b=\pm i\sqrt{C}$

3. $d^2/dx^2\Psi(x) = +C\Psi(x)$, where C is a <u>positive</u> constant.

Possible solutions	d/dx Ψ (x)	$d^2/dx^2\Psi$ (x)	rearrange	eigenvalue? +C =
$\Psi_1(x)$ = A sin(bx) + Bcos(bx) where b is real	Abcos(bx) -Bbsin(bx)	$-Ab^2\sin(bx)$ $-Bb^2\cos(bx)$	$-b^2\ \Psi_1(x)$	$-b^2$ thus $b=\pm\sqrt{-C}$ $b=\pm i\sqrt{C}$
$\Psi_2(x)$ = Aexp(bx) + Bexp(-bx) where b is real	Ab exp(bx) -Bb exp(-bx)	$Ab^2\exp(bx)$ $+Bb^2\exp(-bx)$	$b^2\Psi_1(x)$	b^2 thus $b=\pm\sqrt{C}$