Problem Set 4
On Expectation Values

1. Calculate the mean position $\langle z \rangle$, $\langle z^2 \rangle$, the mean momentum $\langle p_z \rangle$ and the mean kinetic energy $\langle E_{kin} \rangle$ for a physical system of mass m in a state described by the state function

$$\Psi = [2/b]^{1/2} \sin[\pi z/b]$$

where b is a positive constant having the dimensions of length, the system has one degree of freedom, and the integration goes from 0 to b.
You may find it helpful to organize your solution to this problem in the following fashion:

Physical quantity	z	z^2	p_z	E_{kin}
Operator				
OpΨ				
Ψ^*OpΨ				
Integration $\int x\sin^2 ax\,dx = \frac{1}{4}x^2 - (\frac{1}{4}a)x\sin 2ax - (1/8a^2)\cos 2ax$ $\int x^2\sin^2 ax\,dx = (1/6)x^3 - [(\frac{1}{4}a)x^2 - (1/8a^3)]\sin 2ax - (\frac{1}{4}a^2)x\cos 2ax$ $\int \sin^2(ax)\,dx = \frac{1}{2}x - (1/4a)\sin(2ax)$				
Expectation value	$\langle z \rangle =$	$\langle z^2 \rangle =$	$\langle p_z \rangle =$	$\langle E_{kin} \rangle =$

The classical average $\langle z^2 \rangle$ is $(1/3)b^2$
Show that the quantum mechanical expectation value approaches this classical value for this physical system.