Problem Set 6
On Superposition of States

1. Given that ψ_1, ψ_2 ,ψ_3 ,ψ_4 are all the eigenfunctions of an operator **T** with corresponding eigenvalues $a_1 \neq a_2 \neq a_3 \neq a_4$ respectively.
(a) Given that a system is in the state described by

$$\Psi = 0.4\psi_1 + 0.5\psi_2 + c_3\psi_3 + 0.3\psi_4$$

What is the value of c_3?
(b)When the values of T are measured for the system in the state described by Ψ, what fraction of the time will the eigenvalue a_3 be found?
(c)What is the expected average of a series of measurements of T on this system?
(d)What is the expected average of a series of measurements of T^3 on this system?

2. Consider the physical system of a single particle constrained to the line from x = to a (that is, the potential is infinite elsewhere except on this line, and on the line itself V = constant, call it zero.
Suppose that at time t_0, the state of this system is described by the parabolic function

Ψ_{t0}= Nx(a-x) where N is a normalization constant.

(a) If at time t_0, we were to make a measurement of the particle's energy, what would be the possible outcomes of the measurement?
(b) If several copies of this physical system in this state are prepared, what would be the average over all the single measurements at time t_0?
(c) Expand this nonstationary state function Ψ_{t0} = Nx(a-x) in terms of the complete set of energy eigenfunctions $\Psi_{t0} = \sum_n C_n\psi_n$ with coefficients C_n that can be determined.
From this expansion, express the average energy and the probability of each energy measurement outcome in terms of N, a, n. Find N. Make a special note of the results for even values of n and odd values of n.

3. Consider a particle of mass M constrained to move on a circle of radius R where its potential energy is zero. $\Psi_k(\phi) = (1/\sqrt{2\pi})\exp[ik\phi]$ are the eigenfunctions of
$\mathcal{H} = -(\hbar^2/2MR^2)\, d^2/d\phi^2$ and $E = k^2(\hbar^2/2MR^2)$ are the eigenvalues.
The particle is in a physical state that is described by
$F(\phi) = A\{\cos 2\phi + 2\cos 3\phi\}$. *Determine the results of the following sets of experiments on this system, that is, determine the typical outcomes of the experiments, the average values of the results:*
(a) The z component of the angular momentum of the system is measured

<table>
<tr><td rowspan="13">Derivation of predictions here:</td><td>Observed values here:</td></tr>
<tr><td></td></tr>
<tr><td></td></tr>
<tr><td></td></tr>
<tr><td></td></tr>
<tr><td></td></tr>
<tr><td></td></tr>
<tr><td></td></tr>
<tr><td></td></tr>
<tr><td></td></tr>
<tr><td></td></tr>
<tr><td>Average=</td></tr>
</table>

(b) The energy of the system is measured

<table>
<tr><td rowspan="13">Derivation of predictions here:</td><td>Observed values here:</td></tr>
<tr><td></td></tr>
<tr><td></td></tr>
<tr><td></td></tr>
<tr><td></td></tr>
<tr><td></td></tr>
<tr><td></td></tr>
<tr><td></td></tr>
<tr><td></td></tr>
<tr><td></td></tr>
<tr><td></td></tr>
<tr><td>Average=</td></tr>
</table>

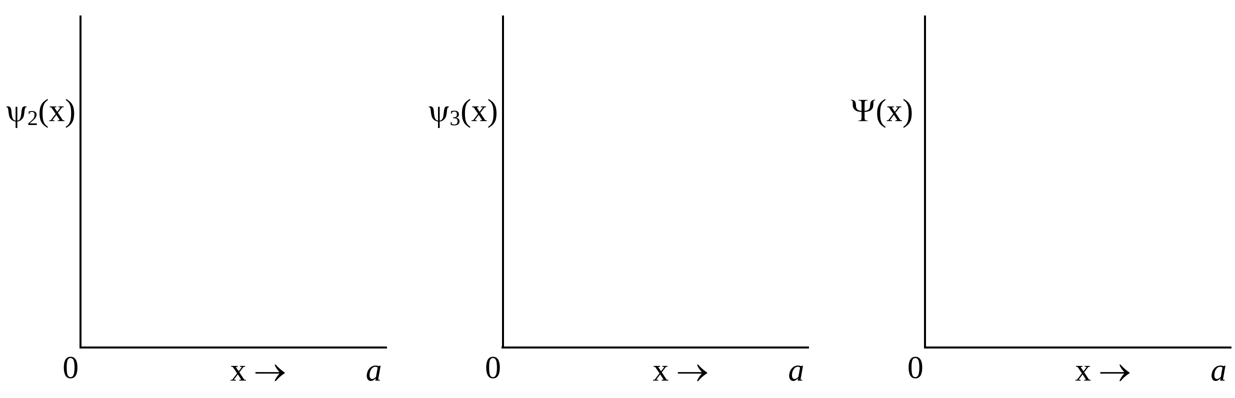
4. A particle of mass *m* in a potential well (with infinitely high walls) in the x dimension is known to be in either the $n = 2$ or $n = 3$ eigenstates with equal probability. The eigenfunctions of these states are $\psi_2(x) = (2/a)^{1/2} \sin [2\pi x/a]$ and $\psi_3(x) = (2/a)^{1/2} \sin [3\pi x/a]$, respectively.

(a) *Write an appropriate wavefunction* Ψ for the system that reflects our knowledge of <u>the state of the system</u>.

(b) *What energies* might be obtained if the energy of the particle is measured?
(c) *Determine the expected average* of a series of measurements of the energy of the particle.
(d) *Write the equation* that shows how the <u>expected</u> <u>mean square deviation</u> of any series of measurements of the energy of the particle can be calculated.
(e) *Carry out the solution* of (d), and then from the final result, determine the expected standard deviation of the series of measurements.
(f) Illustrate a typical table of results from 10 such measurements. *Fill in* the column "Results" . What is the probability of each outcome?

	Result	Deviation		Probability
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
Ave				

(g) *Sketch* (1) $\psi_2(x)$ (2) $\psi_3(x)$ (3) one of the wavefunctions in (a)



(h) Suppose an electron is contained in a two-dimensional potential well (with infinitely high walls) whose shape is that of a rectangular sheet with dimensions $a \times b$. *Write the Schrodinger equation* that needs to be solved for this system.
(i) *Show* that the method of separation of variables may be used to solve this problem, i.e., to find the eigenfunctions and eigenvalues.

(j) Given the results of your proof above, *write down* the possible energy *eigenfunctions* for an electron confined to a sheet with dimensions $a \times b$. Given the results of your proof above, *write down* the corresponding *energy eigenvalues* opposite the eigenfunction