

Problem Set 10 Answers

| | $\alpha\alpha\alpha$ | $\alpha\alpha\beta$ | $\alpha\beta\alpha$ | $\beta\alpha\alpha$ | $\alpha\beta\beta$ | $\beta\alpha\beta$ | $\beta\beta\alpha$ | $\beta\beta\beta$ |
|-------------------------|---|---|--|---|--|---|--|--|
| $\alpha\alpha\alpha$ | $-\frac{1}{2} [v_1 + v_2 + v_3] + \frac{1}{4} [J_{12} + J_{13} + J_{23}]$ | | | | | | | |
| $\alpha\alpha\beta$ | | $-\frac{1}{2} [v_1 + v_2 - v_3] + \frac{1}{4} [J_{12} - J_{13} - J_{23}]$ | $\frac{1}{2} J_{23}$ | $\frac{1}{2} J_{13}$ | | | | |
| $\alpha\beta\alpha$ | | $\frac{1}{2} J_{23}$ | $-\frac{1}{2} [v_1 - v_2 + v_3] + \frac{1}{4} [-J_{12} + J_{13} - J_{23}]$ | $\frac{1}{2} J_{12}$ | | | | |
| $H = \beta\alpha\alpha$ | | $\frac{1}{2} J_{13}$ | $\frac{1}{2} J_{12}$ | $-\frac{1}{2} [-v_1 + v_2 + v_3] + \frac{1}{4} [-J_{12} - J_{13} + J_{23}]$ | | | | |
| $\alpha\beta\beta$ | | | | | $-\frac{1}{2} [v_1 - v_2 - v_3] + \frac{1}{4} [-J_{12} - J_{13} + J_{23}]$ | $\frac{1}{2} J_{12}$ | $\frac{1}{2} J_{13}$ | |
| $\beta\alpha\beta$ | | | | | $\frac{1}{2} J_{12}$ | $-\frac{1}{2} [-v_1 + v_2 - v_3] + \frac{1}{4} [-J_{12} + J_{13} - J_{23}]$ | $\frac{1}{2} J_{23}$ | |
| $\beta\beta\alpha$ | | | | | $\frac{1}{2} J_{13}$ | $\frac{1}{2} J_{23}$ | $-\frac{1}{2} [-v_1 - v_2 + v_3] + \frac{1}{4} [J_{12} - J_{13} - J_{23}]$ | |
| $\beta\beta\beta$ | | | | | | | | $-\frac{1}{2} [-v_1 - v_2 - v_3] + \frac{1}{4} [J_{12} + J_{13} + J_{23}]$ |

| | $\alpha\alpha\alpha$ | $\alpha\alpha\beta$ | $\alpha\beta\alpha$ | $\beta\alpha\alpha$ | $\alpha\beta\beta$ | $\beta\alpha\beta$ | $\beta\beta\alpha$ | $\beta\beta\beta$ |
|----------------------|------------------------------|-----------------------------|------------------------------|-------------------------------|------------------------------|-----------------------------|------------------------------|------------------------------|
| $\alpha\alpha\alpha$ | -90×10^6 -44.275 | | | | | | | |
| $\alpha\alpha\beta$ | | -30×10^6 +4.875 | - 0.65 | +3.7 | | | | |
| $\alpha\beta\alpha$ | | - 0.65 | -30×10^6 - 4.225 | +7.4 | | | | |
| $\beta\alpha\alpha$ | | +3.7 | +7.4 | -30×10^6 - 55.375 | | | | |
| $\alpha\beta\beta$ | | | | | $+30 \times 10^6$ +43.625 | +7.4 | +3.7 | |
| $\beta\alpha\beta$ | | | | | +7.4 | $+30 \times 10^6$ +1.175 | - 0.65 | |
| $\beta\beta\alpha$ | | | | | +3.7 | - 0.65 | $+30 \times 10^6$ - 0.525 | |
| $\beta\beta\beta$ | | | | | | | | $+90 \times 10^6$ +54.725 |

| | Ψ_1 | Ψ_4 | Ψ_3 | Ψ_2 | Ψ_7 | Ψ_6 | Ψ_5 | Ψ_8 |
|----------|------------------------------|-----------------------------|-------------------------------|-------------------------------|-----------------------------------|------------------------------|-------------------------------|------------------------------|
| Ψ_1 | -90×10^6 -44.275 | | | | | | | |
| Ψ_4 | | -30×10^6 +5.106 | | | | | | |
| Ψ_3 | | | -30×10^6 - 3.1778 | | | | | |
| Ψ_2 | | | | -30×10^6 - 56.653 | | | | |
| Ψ_7 | | | | | $+30 \times 10^6$ +45.152 5 | | | |
| Ψ_6 | | | | | | $+30 \times 10^6$ +0.8668 | | |
| Ψ_5 | | | | | | | $+30 \times 10^6$ - 1.7444 | |
| Ψ_8 | | | | | | | | $+90 \times 10^6$ +54.725 |

| | Ψ_1 | Ψ_4 | Ψ_3 | Ψ_2 | Ψ_7 | Ψ_6 | Ψ_5 | Ψ_8 |
|-------|----------------------|----------|----------|----------|----------|----------|----------|----------|
| $C =$ | $\alpha\alpha\alpha$ | 1.0 | | | | | | |
| | $\alpha\alpha\beta$ | | 0.0582 | 0.1412 | 0.9883 | | | |
| | $\alpha\beta\alpha$ | | -0.0234 | 0.9899 | -0.1401 | | | |
| | $\beta\alpha\alpha$ | | 0.9980 | 0.0149 | -0.0609 | | | |
| | $\alpha\beta\beta$ | | | | | 0.0773 | -0.5996 | 0.7965 |
| | $\beta\alpha\beta$ | | | | | 0.1643 | 0.7957 | 0.5830 |
| | $\beta\beta\alpha$ | | | | | 0.9834 | -0.0858 | -0.1600 |
| | $\beta\beta\beta$ | | | | | | | 1.0 |

| Eigenstate | Eigenfunction | Eigenvalues |
|------------|---|-----------------------------------|
| Ψ_1 | $\alpha\alpha\alpha$ | $E_1 = -90 \times 10^6 - 44.275$ |
| Ψ_2 | $0.9883\alpha\alpha\beta - 0.1401\alpha\beta\alpha - 0.0609\beta\alpha\alpha$ | $E_2 = -30 \times 10^6 - 56.653$ |
| Ψ_3 | $0.1412\alpha\alpha\beta + 0.9899\alpha\beta\alpha + 0.0149\beta\alpha\alpha$ | $E_3 = -30 \times 10^6 - 3.1778$ |
| Ψ_4 | $0.0582\alpha\alpha\beta - 0.0234\alpha\beta\alpha + 0.9980\beta\alpha\alpha$ | $E_4 = -30 \times 10^6 + 5.106$ |
| Ψ_5 | $0.7965\alpha\beta\beta + 0.5830\beta\alpha\beta - 0.1600\beta\beta\alpha$ | $E_5 = +30 \times 10^6 - 1.7444$ |
| Ψ_6 | $-0.5996\alpha\beta\beta + 0.7957\beta\alpha\beta - 0.0858\beta\beta\alpha$ | $E_6 = +30 \times 10^6 + 0.8668$ |
| Ψ_7 | $0.0773\alpha\beta\beta + 0.1643\beta\alpha\beta + 0.9834\beta\beta\alpha$ | $E_7 = +30 \times 10^6 + 45.1525$ |
| Ψ_8 | $\beta\beta\beta$ | $E_8 = +90 \times 10^6 + 54.725$ |

| | $\alpha\alpha\alpha$ | $\alpha\alpha\beta$ | $\alpha\beta\alpha$ | $\beta\alpha\alpha$ | $\alpha\beta\beta$ | $\beta\alpha\beta$ | $\beta\beta\alpha$ | $\beta\beta\beta$ |
|----------------------|----------------------|---------------------|---------------------|---------------------|--------------------|--------------------|--------------------|-------------------|
| $\alpha\alpha\alpha$ | | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | | | | |
| $\alpha\alpha\beta$ | $\frac{1}{2}$ | | | | $\frac{1}{2}$ | $\frac{1}{2}$ | 0 | |
| $\alpha\beta\alpha$ | $\frac{1}{2}$ | | | | $\frac{1}{2}$ | 0 | $\frac{1}{2}$ | |
| $\beta\alpha\alpha$ | $\frac{1}{2}$ | | | | 0 | $\frac{1}{2}$ | $\frac{1}{2}$ | |
| $\alpha\beta\beta$ | | $\frac{1}{2}$ | $\frac{1}{2}$ | 0 | | | | |
| $\beta\alpha\beta$ | | $\frac{1}{2}$ | 0 | $\frac{1}{2}$ | | | | |
| $\beta\beta\alpha$ | | 0 | $\frac{1}{2}$ | $\frac{1}{2}$ | | | | |
| $\beta\beta\beta$ | | | | | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | |

| | Ψ_1 | Ψ_2 | Ψ_3 | Ψ_4 | Ψ_5 | Ψ_6 | Ψ_7 | Ψ_8 |
|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| Ψ_1 | | 0.39365 | 0.5729 | 0.51643 | | | | |
| Ψ_2 | 0.39365 | | | | 0.6242 | 0.1233 | 0.010 | |
| Ψ_3 | 0.5729 | | | | 0.4156 | 0.3201 | 0.55068 | |
| Ψ_4 | 0.51643 | | | | 0.24377 | 0.3679 | 0.5674 | |
| Ψ_5 | | 0.6242 | 0.4156 | 0.24377 | | | | 0.6097 |
| Ψ_6 | | 0.1233 | 0.3201 | 0.3679 | | | | 0.055 |
| Ψ_7 | | 0.010 | 0.55068 | 0.5674 | | | | 0.6125 |
| Ψ_8 | | | | | 0.6097 | 0.055 | 0.6125 | |

| Transition | Frequency, Hz = $E_t - E_s$ = 60×10^6 and | Intensity $\{ [F_x]_{st} \}^2$ |
|------------|---|-----------------------------------|
| E_2-E_1 | -12.378 | 0.155 |
| E_3-E_1 | +41.09 | 0.328 |
| E_4-E_1 | +49.38 | 0.267 |
| E_8-E_5 | +56.47 | 0.372 |
| E_8-E_6 | +53.86 | 0.003 |
| E_8-E_7 | +9.575 | 0.375 |
| E_7-E_4 | +40.05 | 0.322 |
| E_7-E_3 | +48.33 | 0.303 |
| E_7-E_2 | +101.80 | 0.000 1 |
| E_6-E_4 | -4.24 | 0.135 |
| E_6-E_3 | +4.04 | 0.102 |
| E_6-E_2 | +57.52 | 0.015 |
| E_5-E_4 | -6.85 | 0.059 |
| E_5-E_3 | +1.43 | 0.173 |
| E_5-E_2 | +54.91 | 0.390 |

Ordered according to frequency:

| Transition | Frequency, Hz = $E_t - E_s$ = 60×10^6 and | Intensity $\{ [F_x]_{st} \}^2$ |
|------------|---|-----------------------------------|
| E_7-E_2 | +101.80 | 0.000 1 |
| E_6-E_2 | +57.52 | 0.015 |
| E_8-E_5 | +56.47 | 0.372 |
| E_5-E_2 | +54.91 | 0.390 |
| E_8-E_6 | +53.86 | 0.003 |
| E_4-E_1 | +49.38 | 0.267 |
| E_7-E_3 | +48.33 | 0.303 |
| E_3-E_1 | +41.09 | 0.328 |
| E_7-E_4 | +40.05 | 0.322 |
| E_8-E_7 | +9.575 | 0.375 |
| E_6-E_3 | +4.04 | 0.102 |
| E_5-E_3 | +1.43 | 0.173 |
| E_6-E_4 | -4.24 | 0.135 |
| E_5-E_4 | -6.85 | 0.059 |
| E_2-E_1 | -12.378 | 0.155 |