

Problem Set 13 answers

1.

$$P_{12} \Psi_{\text{elec space}}(1,2) :$$

$$P_{12} \sigma_g(1) \bullet \sigma_g(2) = + \sigma_g(1) \bullet \sigma_g(2)$$

$$P_{12} \sigma_u(1) \bullet \sigma_u(2) = + \sigma_u(1) \bullet \sigma_u(2)$$

$$P_{12} 1/\sqrt{2} \{ \sigma_g(1) \bullet \sigma_u(2) + \sigma_u(1) \bullet \sigma_g(2) \} = + 1/\sqrt{2} \{ \sigma_g(1) \bullet \sigma_u(2) + \sigma_u(1) \bullet \sigma_g(2) \}$$

$$P_{12} 1/\sqrt{2} \{ \sigma_g(1) \bullet \sigma_u(2) - \sigma_u(1) \bullet \sigma_g(2) \} = - 1/\sqrt{2} \{ \sigma_g(1) \bullet \sigma_u(2) - \sigma_u(1) \bullet \sigma_g(2) \}$$

$$P_{12} \Psi_{\text{elec spin}}(1,2) :$$

$$P_{12} \alpha(1) \bullet \alpha(2) = + \alpha(1) \bullet \alpha(2)$$

$$P_{12} \beta(1) \bullet \beta(2) = + \beta(1) \bullet \beta(2)$$

$$P_{12} 1/\sqrt{2} \{ \alpha(1) \bullet \beta(2) + \beta(1) \bullet \alpha(2) \} = + 1/\sqrt{2} \{ \alpha(1) \bullet \beta(2) + \beta(1) \bullet \alpha(2) \}$$

$$P_{12} 1/\sqrt{2} \{ \alpha(1) \bullet \beta(2) - \beta(1) \bullet \alpha(2) \} = - 1/\sqrt{2} \{ \alpha(1) \bullet \beta(2) - \beta(1) \bullet \alpha(2) \}$$

$$P_{AB} \Psi_{\text{elec space}}(1,2) :$$

$$P_{AB} \sigma_g(1) \bullet \sigma_g(2) = P_{AB} [2 + 2S] [\phi_A(1) \bullet \phi_B(2) + \phi_B(1) \bullet \phi_A(2) + \phi_A(1) \bullet \phi_B(2)]$$

$$= + [2 + 2S] [\phi_A(1) \bullet \phi_B(2) + \phi_B(1) \bullet \phi_A(2) + \phi_A(1) \bullet \phi_A(2) + \phi_B(1) \bullet \phi_B(2)]$$

$$P_{AB} \sigma_u(1) \bullet \sigma_u(2) = P_{AB} [2 + 2S] [-\phi_A(1) \bullet \phi_B(2) - \phi_B(1) \bullet \phi_A(2) + \phi_A(1) \bullet \phi_A(2) + \phi_B(1) \bullet \phi_B(2)]$$

$$= + [2 + 2S] [-\phi_A(1) \bullet \phi_B(2) - \phi_B(1) \bullet \phi_A(2) + \phi_A(1) \bullet \phi_A(2) + \phi_B(1) \bullet \phi_B(2)]$$

$$P_{AB} 1/\sqrt{2} \{ \sigma_g(1) \bullet \sigma_u(2) - \sigma_u(1) \bullet \sigma_g(2) \} = P_{AB} [2(1-S^2)]^{1/2} [-\phi_A(1) \bullet \phi_B(2) + \phi_B(1) \bullet \phi_A(2)]$$

$$= - [2(1-S^2)]^{1/2} [-\phi_A(1) \bullet \phi_B(2) + \phi_B(1) \bullet \phi_A(2)]$$

$$P_{AB} 1/\sqrt{2} \{ \sigma_g(1) \bullet \sigma_u(2) + \sigma_u(1) \bullet \sigma_g(2) \} = P_{AB} [2(1-S^2)]^{1/2} [\phi_A(1) \bullet \phi_A(2) - \phi_B(1) \bullet \phi_B(2)]$$

$$= - [2(1-S^2)]^{1/2} [\phi_A(1) \bullet \phi_A(2) - \phi_B(1) \bullet \phi_B(2)]$$

$$P_{AB} \Psi_{\text{nucl spin}}(A,B) :$$

$$P_{AB} \alpha(A) \bullet \alpha(B) = + \alpha(A) \bullet \alpha(B)$$

$$P_{AB} \beta(A) \bullet \beta(B) = + \beta(A) \bullet \beta(B)$$

$$P_{AB} 1/\sqrt{2} \{ \alpha(A) \bullet \beta(B) + \beta(A) \bullet \alpha(B) \} = + 1/\sqrt{2} \{ \alpha(A) \bullet \beta(B) + \beta(A) \bullet \alpha(B) \}$$

$$P_{AB} 1/\sqrt{2} \{ \alpha(A) \bullet \beta(B) - \beta(A) \bullet \alpha(B) \} = - 1/\sqrt{2} \{ \alpha(A) \bullet \beta(B) - \beta(A) \bullet \alpha(B) \}$$

$$P_{AB} Y_{\text{rot JM}}(\theta, \phi) = (-1)^J Y_{\text{rot JM}}(\theta, \phi)$$

$$P_{AB} \Psi_{\text{vib}}(R_{AB}) = + \Psi_{\text{vib}}(R_{AB})$$

$$P_{AB} F(X_{CM}) \bullet G(Y_{CM}) \bullet D(Z_{CM}) = + F(X_{CM}) \bullet G(Y_{CM}) \bullet D(Z_{CM})$$

$\Psi_{\text{TOTAL}} =$

$\Psi_{\text{elec space}}(1,2) \bullet \Psi_{\text{elec spin}}(1,2) \bullet \Psi_{\text{vib}}(R_{AB}) \bullet Y_{\text{rot JM}}(\theta, \phi) \bullet \Psi_{\text{nucl spin}}(A, B) \bullet F(X_{CM}) \bullet G(Y_{CM})$

$\bullet D(Z_{CM})$

$P_{12} \Psi_{\text{TOTAL}} = -\Psi_{\text{TOTAL}}$ since electrons are fermions

$P_{AB} \Psi_{\text{TOTAL}} = -\Psi_{\text{TOTAL}}$ if nuclei A and B are fermions

$P_{12}\Psi_{\text{TOTAL}} = -\Psi_{\text{TOTAL}}$ since electrons are fermions

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$\Psi_{\text{elec space}}(1,2)$	$\Psi_{\text{elec spin}}(1,2)$	$\Psi_{\text{vib v}}(R_{AB})$	$Y_{\text{rot JM}}(\theta, \phi)$	$\Psi_{\text{nucl spin}}(A,B)$	$F(X_{CM}) \bullet G(Y_{CM})$ $\bullet D(Z_{CM})$
$\sigma_g(1) \bullet \sigma_g(2)$ ground $X^1\Sigma_g^+$ P_{12} sym, P_{AB} sym	$\{\alpha(1) \bullet \beta(2) - \beta(1) \bullet \alpha(2)\}/\sqrt{2}$ P_{12} antisym	any v P_{AB} sym	$J = \text{even}$ P_{AB} sym	$\{\alpha(A) \bullet \beta(B) - \beta(A) \bullet \alpha(B)\}/\sqrt{2}$ P_{AB} antisym	any n_x, n_y, n_z
$\sigma_g(1) \bullet \sigma_g(2)$ ground $X^1\Sigma_g^+$ P_{12} sym, P_{AB} sym	$\{\alpha(1) \bullet \beta(2) - \beta(1) \bullet \alpha(2)\}/\sqrt{2}$ P_{12} antisym	any v P_{AB} sym	$J = \text{odd}$ P_{AB} antisym	$\alpha(A) \bullet \alpha(B)$ $\beta(A) \bullet \beta(B)$ P_{AB} sym $\{\alpha(A) \bullet \beta(B) + \beta(A) \bullet \alpha(B)\}/\sqrt{2}$	any n_x, n_y, n_z
$\{\sigma_g(1) \bullet \sigma_u(2) - \sigma_u(1) \bullet \sigma_g(2)\}/\sqrt{2}$ $b^3\Sigma_u^+$ P_{12} antisym, P_{AB} antisym	$\alpha(1) \bullet \alpha(2)$ $\beta(1) \bullet \beta(2)$ P_{12} sym $\{\alpha(1) \bullet \beta(2) + \beta(1) \bullet \alpha(2)\}/\sqrt{2}$	any v P_{AB} sym	$J = \text{even}$ P_{AB} sym	$\alpha(A) \bullet \alpha(B)$ $\beta(A) \bullet \beta(B)$ P_{AB} sym $\{\alpha(A) \bullet \beta(B) + \beta(A) \bullet \alpha(B)\}/\sqrt{2}$	any n_x, n_y, n_z
$\{\sigma_g(1) \bullet \sigma_u(2) - \sigma_u(1) \bullet \sigma_g(2)\}/\sqrt{2}$ $b^3\Sigma_u^+$ P_{12} antisym, P_{AB} antisym	$\alpha(1) \bullet \alpha(2)$ $\beta(1) \bullet \beta(2)$ P_{12} sym $\{\alpha(1) \bullet \beta(2) + \beta(1) \bullet \alpha(2)\}/\sqrt{2}$	any v P_{AB} sym	$J = \text{odd}$ P_{AB} antisym	$\{\alpha(A) \bullet \beta(B) - \beta(A) \bullet \alpha(B)\}/\sqrt{2}$ P_{AB} antisym	any n_x, n_y, n_z
$\{\sigma_g(1) \bullet \sigma_u(2) + \sigma_u(1) \bullet \sigma_g(2)\}/\sqrt{2}$ $B^1\Sigma_u^+$ P_{12} sym, P_{AB} antisym	$\{\alpha(1) \bullet \beta(2) - \beta(1) \bullet \alpha(2)\}/\sqrt{2}$ P_{12} antisym	any v P_{AB} sym	$J = \text{even}$ P_{AB} sym	$\alpha(A) \bullet \alpha(B)$ $\beta(A) \bullet \beta(B)$ P_{AB} sym $\{\alpha(A) \bullet \beta(B) + \beta(A) \bullet \alpha(B)\}/\sqrt{2}$	any n_x, n_y, n_z
$\{\sigma_g(1) \bullet \sigma_u(2) + \sigma_u(1) \bullet \sigma_g(2)\}/\sqrt{2}$ $B^1\Sigma_u^+$ P_{12} sym, P_{AB} antisym	$\{\alpha(1) \bullet \beta(2) - \beta(1) \bullet \alpha(2)\}/\sqrt{2}$ P_{12} antisym	any v P_{AB} sym	$J = \text{odd}$ P_{AB} antisym	$\{\alpha(A) \bullet \beta(B) - \beta(A) \bullet \alpha(B)\}/\sqrt{2}$ P_{AB} antisym	any n_x, n_y, n_z
$\sigma_u(1) \bullet \sigma_u(2)$ very excited ${}^1\Sigma_g^+$ P_{12} sym, P_{AB} sym	$\{\alpha(1) \bullet \beta(2) - \beta(1) \bullet \alpha(2)\}/\sqrt{2}$ P_{12} antisym	any v P_{AB} sym	$J = \text{even}$ P_{AB} sym	$\{\alpha(A) \bullet \beta(B) - \beta(A) \bullet \alpha(B)\}/\sqrt{2}$ P_{AB} antisym	any n_x, n_y, n_z
$\sigma_u(1) \bullet \sigma_u(2)$ ${}^1\Sigma_g^+$ very excited state P_{12} sym, P_{AB} sym	$\{\alpha(1) \bullet \beta(2) - \beta(1) \bullet \alpha(2)\}/\sqrt{2}$ P_{12} antisym	any v P_{AB} sym	$J = \text{odd}$ P_{AB} antisym	$\alpha(A) \bullet \alpha(B)$ $\beta(A) \bullet \beta(B)$ P_{AB} sym $\{\alpha(A) \bullet \beta(B) + \beta(A) \bullet \alpha(B)\}/\sqrt{2}$	any n_x, n_y, n_z

Pauli principle applied to heavy hydrogen, D_2 molecule:

Symmetry with respect to P_{12} is the same as derived for H_2 molecule since electronic functions are unchanged.

Symmetry with respect to P_{AB} is the same for the rotational wavefunction, since only the mass change affects this.

Nuclear spin functions are different because spin of deuteron is 1, hence it is a boson and the number of spin functions is different since $m_I = -1, 0, +1$ are possible for $I=1$.

Furthermore, $P_{AB}\Psi_{\text{TOTAL}} = -\Psi_{\text{TOTAL}}$ if nuclei A and B are bosons (integer spin)

Now consider the nuclear spin of nuclei A and B, the two deuterons. The deuteron has spin $I=1$. We can form the eigenfunctions of square of nuclear spin angular momentum \mathbf{I}^2 from the spin functions that are already eigenfunctions of \mathbf{I}_z .

Eigenfunctions of \mathbf{I}_z	M_I		I_{TOT}	Eigenfunctions of \mathbf{I}^2	P_{AB}
$\alpha(A)\bullet\alpha(B)$	$+1 + 1 = 2$	degenerate	2	$\alpha(A)\bullet\alpha(B)$	+
$\beta(A)\bullet\beta(B)$	$-1 - 1 = -2$		2	$\beta(A)\bullet\beta(B)$	+
$\alpha(A)\bullet\beta(B)$	$+1 - 1 = 0$		2	$1/\sqrt{2}\{\alpha(A)\bullet\beta(B) + \beta(A)\bullet\alpha(B)\}$	+
$\beta(A)\bullet\alpha(B)$	$-1 + 1 = 0$		1	$1/\sqrt{2}\{\alpha(A)\bullet\beta(B) - \beta(A)\bullet\alpha(B)\}$	-
$\gamma(A)\bullet\gamma(B)$	$0 + 0 = 0$		0	$\gamma(A)\bullet\gamma(B)$	+
$\alpha(A)\bullet\gamma(B)$	$+1 + 0 = 1$		2	$1/\sqrt{2}\{\alpha(A)\bullet\gamma(B) + \gamma(A)\bullet\alpha(B)\}$	+
$\gamma(A)\bullet\alpha(B)$	$0 + 1 = 1$		1	$1/\sqrt{2}\{\alpha(A)\bullet\gamma(B) - \gamma(A)\bullet\alpha(B)\}$	-
$\beta(A)\bullet\gamma(B)$	$-1 + 0 = -1$		2	$1/\sqrt{2}\{\beta(A)\bullet\gamma(B) + \gamma(A)\bullet\beta(B)\}$	+
$\gamma(A)\bullet\beta(B)$	$0 - 1 = -1$		1	$1/\sqrt{2}\{\beta(A)\bullet\gamma(B) - \gamma(A)\bullet\beta(B)\}$	-

Classified the above 9 functions that are eigenfunctions of I_{TOT}^2 according to 'symmetric' or 'antisymmetric' with respect to interchange of deuterons A and B, shown respectively as (+) or (-).

$P_{AB}\Psi_{\text{nucl spin}}(A,B)$:

$$P_{AB}\alpha(A)\bullet\alpha(B) = +\alpha(A)\bullet\alpha(B)$$

$$P_{AB}\beta(A)\bullet\beta(B) = +\beta(A)\bullet\beta(B)$$

$$P_{AB}\gamma(A)\bullet\gamma(B) = +\gamma(A)\bullet\gamma(B)$$

$$P_{AB}1/\sqrt{2}\{\alpha(A)\bullet\beta(B) + \beta(A)\bullet\alpha(B)\} = +1/\sqrt{2}\{\alpha(A)\bullet\beta(B) + \beta(A)\bullet\alpha(B)\}$$

$$P_{AB}1/\sqrt{2}\{\alpha(A)\bullet\gamma(B) + \gamma(A)\bullet\alpha(B)\} = +1/\sqrt{2}\{\alpha(A)\bullet\gamma(B) + \gamma(A)\bullet\alpha(B)\}$$

$$P_{AB}1/\sqrt{2}\{\gamma(A)\bullet\beta(B) + \beta(A)\bullet\gamma(B)\} = +1/\sqrt{2}\{\gamma(A)\bullet\beta(B) + \beta(A)\bullet\gamma(B)\}$$

$$P_{AB}1/\sqrt{2}\{\alpha(A)\bullet\beta(B) - \beta(A)\bullet\alpha(B)\} = -1/\sqrt{2}\{\alpha(A)\bullet\beta(B) - \beta(A)\bullet\alpha(B)\}$$

$$P_{AB}1/\sqrt{2}\{\alpha(A)\bullet\gamma(B) - \gamma(A)\bullet\alpha(B)\} = -1/\sqrt{2}\{\alpha(A)\bullet\gamma(B) - \gamma(A)\bullet\alpha(B)\}$$

$$P_{AB}1/\sqrt{2}\{\gamma(A)\bullet\beta(B) - \beta(A)\bullet\gamma(B)\} = -1/\sqrt{2}\{\gamma(A)\bullet\beta(B) - \beta(A)\bullet\gamma(B)\}$$

$P_{12}\Psi_{\text{TOTAL}} = -\Psi_{\text{TOTAL}}$ since electrons are fermions

$P_{AB}\Psi_{\text{TOTAL}} = +\Psi_{\text{TOTAL}}$ if nuclei A and B are bosons

$P_{12}\Psi_{\text{TOTAL}} = -\Psi_{\text{TOTAL}}$ since electrons are fermions

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$\Psi_{\text{elec space}}(1,2)$	$\Psi_{\text{elec spin}}(1,2)$	$\Psi_{\text{vib}}_v(R_{AB})$	$Y_{\text{rot}}_{JM}(\theta, \phi)$	$\Psi_{\text{nucl spin}}(A,B)$	$F \bullet G \bullet D$ R_{CM}
$\sigma_g(1) \bullet \sigma_g(2)$ ground $X^1\Sigma_g^+$ P_{12} sym, P_{AB} sym	$\{\alpha(1) \bullet \beta(2) - \beta(1) \bullet \alpha(2)\}/\sqrt{2}$ P_{12} antisym	any v P_{AB} sym	$J = \text{even}$ P_{AB} sym	$\alpha(A) \bullet \alpha(B) \quad \beta(A) \bullet \beta(B)$ $\gamma(A) \bullet \gamma(B) \quad P_{AB}$ sym $\{\alpha(A) \bullet \beta(B) + \beta(A) \bullet \alpha(B)\}/\sqrt{2}$ $\{\alpha(A) \bullet \gamma(B) + \gamma(A) \bullet \alpha(B)\}/\sqrt{2}$ $\{\gamma(A) \bullet \beta(B) + \beta(A) \bullet \gamma(B)\}/\sqrt{2}$	any n_x, n_y, n_z
$\sigma_g(1) \bullet \sigma_g(2)$ ground $X^1\Sigma_g^+$ P_{12} sym, P_{AB} sym	$\{\alpha(1) \bullet \beta(2) - \beta(1) \bullet \alpha(2)\}/\sqrt{2}$ P_{12} antisym	any v P_{AB} sym	$J = \text{odd}$ P_{AB} antisym	$\{\alpha(A) \bullet \beta(B) - \beta(A) \bullet \alpha(B)\}/\sqrt{2}$ $\{\alpha(A) \bullet \gamma(B) - \gamma(A) \bullet \alpha(B)\}/\sqrt{2}$ $\{\gamma(A) \bullet \beta(B) - \beta(A) \bullet \gamma(B)\}/\sqrt{2}$ P_{AB} antisym	any n_x, n_y, n_z
$\{\sigma_g(1) \bullet \sigma_u(2) - \sigma_u(1) \bullet \sigma_g(2)\}/\sqrt{2}$ $b^3\Sigma_u^+$ P_{12} antisym, P_{AB} antisym	$\alpha(1) \bullet \alpha(2)$ $\beta(1) \bullet \beta(2)$ $\{\alpha(1) \bullet \beta(2) + \beta(1) \bullet \alpha(2)\}/\sqrt{2}$ P_{12} sym	any v P_{AB} sym	$J = \text{even}$ P_{AB} sym	$\{\alpha(A) \bullet \beta(B) - \beta(A) \bullet \alpha(B)\}/\sqrt{2}$ $\{\alpha(A) \bullet \gamma(B) - \gamma(A) \bullet \alpha(B)\}/\sqrt{2}$ $\{\gamma(A) \bullet \beta(B) - \beta(A) \bullet \gamma(B)\}/\sqrt{2}$ P_{AB} antisym	any n_x, n_y, n_z
$\{\sigma_g(1) \bullet \sigma_u(2) - \sigma_u(1) \bullet \sigma_g(2)\}/\sqrt{2}$ $b^3\Sigma_u^+$ P_{12} antisym, P_{AB} antisym	$\alpha(1) \bullet \alpha(2)$ $\beta(1) \bullet \beta(2)$ $\{\alpha(1) \bullet \beta(2) + \beta(1) \bullet \alpha(2)\}/\sqrt{2}$ P_{12} sym	any v P_{AB} sym	$J = \text{odd}$ P_{AB} antisym	$\alpha(A) \bullet \alpha(B) \quad \beta(A) \bullet \beta(B)$ $\gamma(A) \bullet \gamma(B) \quad P_{AB}$ sym $\{\alpha(A) \bullet \beta(B) + \beta(A) \bullet \alpha(B)\}/\sqrt{2}$ $\{\alpha(A) \bullet \gamma(B) + \gamma(A) \bullet \alpha(B)\}/\sqrt{2}$ $\{\gamma(A) \bullet \beta(B) + \beta(A) \bullet \gamma(B)\}/\sqrt{2}$	any n_x, n_y, n_z
$\{\sigma_g(1) \bullet \sigma_u(2) + \sigma_u(1) \bullet \sigma_g(2)\}/\sqrt{2}$ $B^1\Sigma_u^+$ P_{12} sym, P_{AB} antisym	$\{\alpha(1) \bullet \beta(2) - \beta(1) \bullet \alpha(2)\}/\sqrt{2}$ P_{12} antisym	any v P_{AB} sym	$J = \text{even}$ P_{AB} sym	$\{\alpha(A) \bullet \beta(B) - \beta(A) \bullet \alpha(B)\}/\sqrt{2}$ $\{\alpha(A) \bullet \gamma(B) - \gamma(A) \bullet \alpha(B)\}/\sqrt{2}$ $\{\gamma(A) \bullet \beta(B) - \beta(A) \bullet \gamma(B)\}/\sqrt{2}$ P_{AB} antisym	any n_x, n_y, n_z
$\{\sigma_g(1) \bullet \sigma_u(2) + \sigma_u(1) \bullet \sigma_g(2)\}/\sqrt{2}$ $B^1\Sigma_u^+$ P_{12} sym, P_{AB} antisym	$\{\alpha(1) \bullet \beta(2) - \beta(1) \bullet \alpha(2)\}/\sqrt{2}$ P_{12} antisym	any v P_{AB} sym	$J = \text{odd}$ P_{AB} antisym	$\alpha(A) \bullet \alpha(B) \quad \beta(A) \bullet \beta(B)$ $\gamma(A) \bullet \gamma(B) \quad P_{AB}$ sym $\{\alpha(A) \bullet \beta(B) + \beta(A) \bullet \alpha(B)\}/\sqrt{2}$ $\{\alpha(A) \bullet \gamma(B) + \gamma(A) \bullet \alpha(B)\}/\sqrt{2}$ $\{\gamma(A) \bullet \beta(B) + \beta(A) \bullet \gamma(B)\}/\sqrt{2}$	any n_x, n_y, n_z
$\sigma_u(1) \bullet \sigma_u(2)$ very excited ${}^1\Sigma_g^+$ P_{12} sym, P_{AB} sym	$\{\alpha(1) \bullet \beta(2) - \beta(1) \bullet \alpha(2)\}/\sqrt{2}$ P_{12} antisym	any v P_{AB} sym	$J = \text{even}$ P_{AB} sym	$\alpha(A) \bullet \alpha(B) \quad \beta(A) \bullet \beta(B)$ $\gamma(A) \bullet \gamma(B) \quad P_{AB}$ sym $\{\alpha(A) \bullet \beta(B) + \beta(A) \bullet \alpha(B)\}/\sqrt{2}$ $\{\alpha(A) \bullet \gamma(B) + \gamma(A) \bullet \alpha(B)\}/\sqrt{2}$ $\{\gamma(A) \bullet \beta(B) + \beta(A) \bullet \gamma(B)\}/\sqrt{2}$	any n_x, n_y, n_z
$\sigma_u(1) \bullet \sigma_u(2)$ ${}^1\Sigma_g^+$ very excited state P_{12} sym, P_{AB} sym	$\{\alpha(1) \bullet \beta(2) - \beta(1) \bullet \alpha(2)\}/\sqrt{2}$ P_{12} antisym	any v P_{AB} sym	$J = \text{odd}$ P_{AB} antisym	$\{\alpha(A) \bullet \beta(B) - \beta(A) \bullet \alpha(B)\}/\sqrt{2}$ $\{\alpha(A) \bullet \gamma(B) - \gamma(A) \bullet \alpha(B)\}/\sqrt{2}$ $\{\gamma(A) \bullet \beta(B) - \beta(A) \bullet \gamma(B)\}/\sqrt{2}$ P_{AB} antisym	any n_x, n_y, n_z

2. with the plane of the paper as the xz plane and z axis along line of centers:

			approximate MO (using the sign convention of z axes pointing towards each other)
(a)	CO	$\pi 2p_y$	$[2p_y(C) + 2p_y(O)]$
(b)	F ₂	$\pi^*_g 2p_x$	$[2p_x(F_1) - 2p_x(F_2)]$
(c)	F ₂	$\pi_u 2p_x$	$[2p_x(F_1) + 2p_x(F_2)]$
(d)	CO	$\sigma^* 2p_z$	$[2p_z(C) - 2p_z(O)]$
(e)	CO	$\pi^* 2p_y$	$[2p_y(C) - 2p_y(O)]$
(f)	O ₂	$\pi_u 2p_y$	$[2p_y(O) + 2p_y(O)]$
(g)	HF	σ	$[1s(H) + 2s(F)]$
(h)	CO	$\pi 2p_x$	$[2p_x(C) + 2p_x(O)]$
(i)	NO	$\sigma 2p_z$	$[2p_z(N) + 2p_z(O)]$
(j)	N ₂	$\sigma^*_u 2s$	$[2s(N_1) - 2s(N_2)]$
(k)	CO	$\sigma 2s$	$2s(O)$
(l)	F ₂	$\sigma_g 2s$	$[2s(F_1) + 2s(F_2)]$
(m)	F ₂	$\sigma_g 2p_z$	$[2p_z(F_1) + 2p_z(F_2)]$
(n)	CO	$\pi^* 2p_x$	$[2p_x(C) - 2p_x(O)]$
(o)	N ₂	$\sigma^*_u 2p_z$	$[2p_z(N_1) - 2p_z(N_2)]$
(p)	CO	σ	$[2s(C) + 2p_z(O)]$
(q)	F ₂	$\sigma^*_u 2p_z$	$[2p_z(F_1) - 2p_z(F_2)]$
(r)	HF	σ	$[1s(H) + 2p_z(F)]$

3. 19 electrons on K: $1s^2 2s^2 2p^6 3s^2 3p^6 4s$

The inner shells remain nearly atomic and we consider them as core, largely undisturbed by bonding. So we have two approximately one-electron H-atom like atoms:

$4s(K_1) + 4s(K_2)$ would give a molecular orbital $\sigma_g 4s$

Consider the angular momenta of the electrons: $s=1/2$ and $s=1/2$ can give rise to $S=0$ or $S=1$ four combinations altogether, just as for H₂ molecule in part 1 of this problem set. Following the steps in part 1:

The two electrons will have the electronic configuration ($\sigma_g 4s$)² leading to the ground electronic state $X^1\Sigma_g^+$ just as shpwn in the figure from the paper from *J. Mol. Spectros.*

$\sigma_g(1)\bullet\sigma_g(2)$ ground $X^1\Sigma_g^+$ P_{12} sym, P_{AB} sym	$\{\alpha(1)\bullet\beta(2) - \beta(1)\bullet\alpha(2)\}/\sqrt{2}$ P_{12} antisym	vibrational states any v P_{AB} sym	rotational states $J = \text{odd}$ P_{AB} antisym	nuclear spin states $\{\alpha(A)\bullet\beta(B) - \beta(A)\bullet\alpha(B)\}/\sqrt{2}$ $\{\alpha(A)\bullet\gamma(B) - \gamma(A)\bullet\alpha(B)\}/\sqrt{2}$ $\{\gamma(A)\bullet\beta(B) - \beta(A)\bullet\gamma(B)\}/\sqrt{2}$ P_{AB} antisym
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Just as found for H₂ in part 1, we will find, just using just the MO σ_g 4s for each K, the other electronic excited states $^3\Sigma_u^+$, $^1\Sigma_u^+$, $^1\Sigma_g^+$ just as given in the table in part 1, except that, rather than a 1s orbital, we have a 4s orbital on the K atom. We do indeed find the A $^1\Sigma_u^+$ and $^1\Sigma_g^+$ in the J. Mol. Spectrosc. figure. The $^3\Sigma_u^+$ state is not shown, although that also must exist.

Where do the other states $b^3\Pi_g$, $B^1\Pi_u$, $^1\Pi_g$, $^1\Delta_g$ come from? Unlike H atom which has no orbitals p or d in the valence shell, K does have relatively low-lying 4p and 3d atomic orbitals it can use to form molecular orbitals. By following the same procedure as in part 1, but using a 4s orbital on the first K atom and a 4p orbital on the second K atom, we will find the combinations that give rise to $b^3\Pi_g$, $B^1\Pi_u$, $^1\Pi_g$, and $^1\Delta_g$. Look at $l_1=0$ for the 4s and $l_2=1$ for the 4p on the second K atom are associated with $m_1=0$ and $m_2=0, \pm 1$. Right away we can see that for the MO, $\lambda = 0, \pm 1$ can arise from these combinations of m_1 and m_2 . And from $\lambda = 0, \pm 1$, the sums of λ could be 0 or 1 or 2. which are Σ , Π , and Δ states respectively. It now only requires doing the combinations, just as we did in part 1 for 1s and 1s, we need to do for 4s and 4p to find out when you can have g or u, S=0 (singlet) or 1 (triplet) combinations by using the fact that electrons are fermions.