*Name*____

Chemistry 344

Exam I

Monday September 24, 2001

2:00 -2:50 PM

NO CALCULATORS PERMITTED

1. A particle of mass m in a potential well (with infinitely high walls) in the x dimension is known to be in either the $n = 2$ or $n = 3$ eigenstates with equal probability. The eigenfunctions of these states are $\psi_2(x) = (2/a)^{1/2} \sin [2\pi x/a]$ and $\psi_3(x) = (2/a)^{1/2} \sin [3\pi x/a]$, respectively.

(a) *Write an appropriate wavefunction* Ψ for the system that reflects our knowledge of the state of the system.

(b) *Prove* that $\psi_2(x) = (2/a)^{1/2} \sin [2\pi x/a]$ is an eigenfunction of the Hamiltonian operator for this particle.

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(c) *What energies* might be obtained if the energy of the particle is measured?

(d) *Determine the expected average* of a series of measurements of the energy of the particle.

(e) *Which postulate* predicts the standard deviation of such a series of measurements? State it.

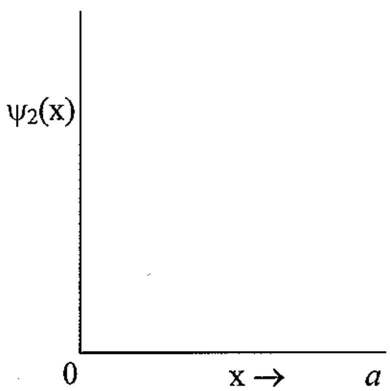
Write the equation that shows how the expected mean square deviation of any series of measurements of the energy of the particle can be calculated.

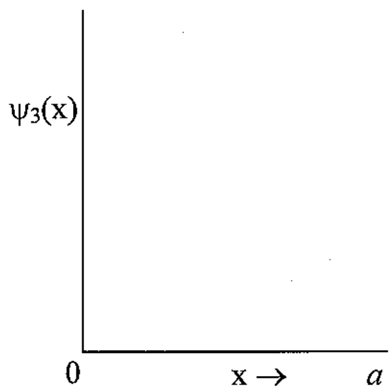
(f) *Carry out the solution* of (e), and then from the final result, determine the expected standard deviation of the series of measurements.

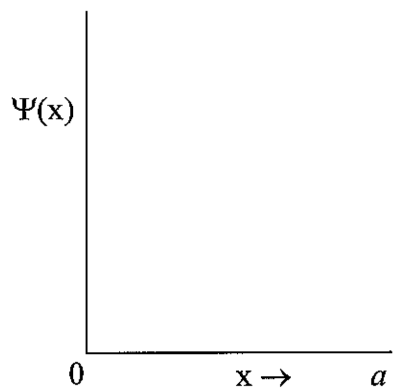
(e) Illustrate a typical table of results from 10 such measurements. *Fill in* the column "Results" . What is the probability of each outcome?

	Result	Deviation		Probability
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
Ave				

(g) *Sketch* (1) $\psi_2(x)$ (2) $\psi_3(x)$ (3) one of the wavefunctions in (a)







(h) Suppose an electron is contained in a two-dimensional potential well (with infinitely high walls) whose shape is that of a rectangular sheet with dimensions $a \times b$. *Write the Schrodinger equation* that needs to be solved for this system.

(i) *Show* that the method of separation of variables may be used to solve this problem, i.e., to find the eigenfunctions and eigenvalues.

(j) Given the results of your proof above, *write down* the possible energy *eigenfunctions* for an electron confined to a sheet with dimensions $a \times b$. Given the results of your proof above, *write down* the corresponding *energy eigenvalues* opposite the eigenfunction

List of possibly useful integrals that will be provided with each exam

$\int \sin(ax)dx = -(1/a)\cos(ax)$

$\int \cos(ax)dx = (1/a)\sin(ax)$

$\int \sin^2(ax)dx = ½\, x - (1/4a)\sin(2ax)$

$\int \cos^2(ax)dx = ½\, x + (1/4a)\sin(2ax)$

$\int \sin(ax)\sin(bx)dx = [1/2(a-b)]\sin[(a-b)x)] - [1/2(a+b)]\sin[(a+b)x)],\quad a^2 \neq b^2$

$\int \cos(ax)\cos(bx)dx = [1/2(a-b)]\sin[(a-b)x)] + [1/2(a+b)]\sin[(a+b)x)],\quad a^2 \neq b^2$

$\int x \sin(ax)dx = (1/a^2)\sin(ax) - (x/a)\cos(ax)$

$\int x \cos(ax)dx = (1/a^2)\cos(ax) + (x/a)\sin(ax)$

$\int x^2 \cos(ax)dx = [(a^2x^2 - 2)/a^3]\sin(ax) + 2x\cos(ax)/a^2$

$\int x^2 \sin(ax)dx = -[(a^2x^2 - 2)/a^3]\cos(ax) + 2x\sin(ax)/a^2$

$\int x \sin^2(ax)dx = x^2/4 - x\sin(2ax)/4a - \cos(2ax)/8a^2$

$\int x^2 \sin^2(ax)dx = x^3/6 - [x^2/4a - 1/8a^3]\sin(2ax) - x\cos(2ax)/4a^2$

$\int x \cos^2(ax)dx = x^2/4 + x\sin(2ax)/4a + \cos(2ax)/8a^2$

$\int x^2 \cos^2(ax)dx = x^3/6 + [x^2/4a - 1/8a^3]\sin(2ax) + x\cos(2ax)/4a^2$

$\int x \exp(ax)dx = \exp(ax)\,(ax-1)/a^2$

$\int x \exp(-ax)dx = \exp(-ax)\,(-ax-1)/a^2$

$\int x^2 \exp(ax)dx = \exp(ax)\,[x^2/a - 2x/a^2 + 2/a^3]$

$\int x^m \exp(ax)dx = \exp(ax) \sum_{r=0 \text{ to } m} (-1)^r\, m!x^{m-r}/(m-r)!a^{r+1}$

$\int_0^\infty x^n \exp(-ax)dx = n!/a^{n+1}$ $\quad a > 0$, n positive integer

$\int_0^\infty x^2 \exp(-ax^2)dx = (1/4a)(\pi/a)^{½}$ $\quad a > 0$

$\int_0^\infty x^{2n} \exp(-ax^2)dx = (1\cdot3\cdot5\cdot\ldots\cdot(2n-1)/(2^{n+1}a^n)(\pi/a)^{½}$ $\quad a > 0$

$\int_0^\infty x^{2n+1} \exp(-ax^2)dx = n!/2a^{n+1}$ $\quad a > 0$, n positive integer

$\int_0^\infty \exp(-a^2x^2)dx = (1/2a)(\pi)^{½}$ $\quad a > 0$

$\int_0^\infty \exp(-ax)\cos(bx)dx = a/(a^2+b^2)$ $\quad a > 0$

$\int_0^\infty \exp(-ax)\sin(bx)dx = b/(a^2+b^2)$ $\quad a > 0$

$\int_0^\infty x\exp(-ax)\sin(bx)dx = 2ab/(a^2+b^2)^2$ $\quad a > 0$

$\int_0^\infty x\exp(-ax)\cos(bx)dx = (a^2-b^2)/(a^2+b^2)^2$ $\quad a > 0$

$\int_0^\infty \exp(-a^2x^2)\cos(bx)dx = [(\pi)^{½}/2a]\cdot\exp[-b^2/4a^2]$ $\quad ab \neq 0$