CHEMISTRY 542

Exam II
November 8, 2004

In applying the principles of Quantum Mechanics in answering each question, be sure to *state the principle you are using at each step*.

1. Given the complete orthonormal set of functions $\{\alpha, \beta, \gamma\}$ which are eigenfunctions of the z component of angular momentum I_z with eigenvalues $\hbar$, $-\hbar$, and 0 respectively. The operators I_- and I_+ have the following properties:

$I_-\alpha = \sqrt{2}\,\hbar\,\gamma \qquad I_+\alpha = 0$

$I_-\gamma = \sqrt{2}\,\hbar\,\beta \qquad I_+\gamma = \sqrt{2}\,\hbar\,\alpha$

$I_-\beta = 0 \qquad I_+\beta = \sqrt{2}\,\hbar\,\gamma$

$I_x = (I_+ + I_-)/2 \qquad I_y = (I_+ - I_-)/2i$

(a) Find the matrix representation of the operator I_+, I_-, I_z, I_x, and I_y in this basis set.

I_+ =

I_- =

I_z =

I_x =

I_y =

(b) Find the eigenvalues of the I_x operator.

(c) Find the eigenfunctions of the I_x operator.

(d) Suppose it is found that the system is described by the state function:

$$\Psi = 2^{-1/2}\alpha + 1/2\gamma + 1/2\beta$$

What values would result from a measurement of I_z on this system?

What is the expected average of a series of measurements of I_x on a system described by Ψ ?

2. The matrix representations of x, x^2 and x^4 in the basis of the complete orthonormal set of harmonic oscillator eigenfunctions $\{\varphi_0, \varphi_1, \varphi_2, \varphi_3, \ldots\}$ are given by: (where $a = \hbar/4\pi\nu_e\mu$), and the corresponding energy eigenvalues are $(v+\frac{1}{2})\hbar\nu_e$

$x = a^{1/2}$

0	$\sqrt{1}$	0	0	0	...
$\sqrt{1}$	0	$\sqrt{2}$	0	0	...
0	$\sqrt{2}$	0	$\sqrt{3}$	0	...
0	0	$\sqrt{3}$	0	$\sqrt{4}$	...
...	...	...	...	...	...

$x^2 = a$

1	0	$\sqrt{2}$	0	0	0	0	0	...
0	3	0	$\sqrt{6}$	0	0	0	0	...
$\sqrt{2}$	0	5	0	$\sqrt{12}$	0	0	0	...
0	$\sqrt{6}$	0	7	0	$\sqrt{20}$	0	0	...
0	0	$\sqrt{12}$	0	9	0	$\sqrt{30}$	0	...
...	...	...	...	...	...	...	...	...

$x^4 = a^2$

3	0	$6\sqrt{2}$	0	$\sqrt{24}$	0	0	0	...
0	15	0	$10\sqrt{6}$	0	$\sqrt{120}$	0	0	...
$6\sqrt{2}$	0	39	0	$14\sqrt{12}$	0	$\sqrt{360}$	0	...
0	$10\sqrt{6}$	0	75	0	$18\sqrt{20}$	0	$\sqrt{840}$	...
$\sqrt{24}$	0	$14\sqrt{12}$	0	123	0	$\sqrt{1680}$	0	...
...	...	...	...	...	...	...	...	...

Suppose a harmonic oscillator is placed in an electric field, i.e., perturbed by $h = c\,x$ where c is a constant,
what is the energy of the v=2 level in the presence of the perturbation? Provide an answer that is correct to second order.

What is the wavefunction for this level, correct to first order?

3. Using the complete orthonormal basis set $\{ \varphi_1 , \varphi_2 , \varphi_3 , \varphi_4 \}$, the **H** matrix for a physical system is given by:

H =

-7	-3	0	0
-3	-9	0	0
0	0	-7	-7
0	0	-7	-9

Find the energy eigenvalues and the eigenfunctions of this system.