

Chemistry 543

Spring 1999 Final Exam
May 6, 1999 10:00 AM -1:00 PM

1. The laser excitation spectrum near the head of the ($v' = 0, v'' = 4$) band of the $X^1\Sigma_g^+ \rightarrow A(0_u^+)$ system of the Bi₂ molecule is shown below. [*J. Mol. Spectrosc.* 194, 1-7 (1999)].

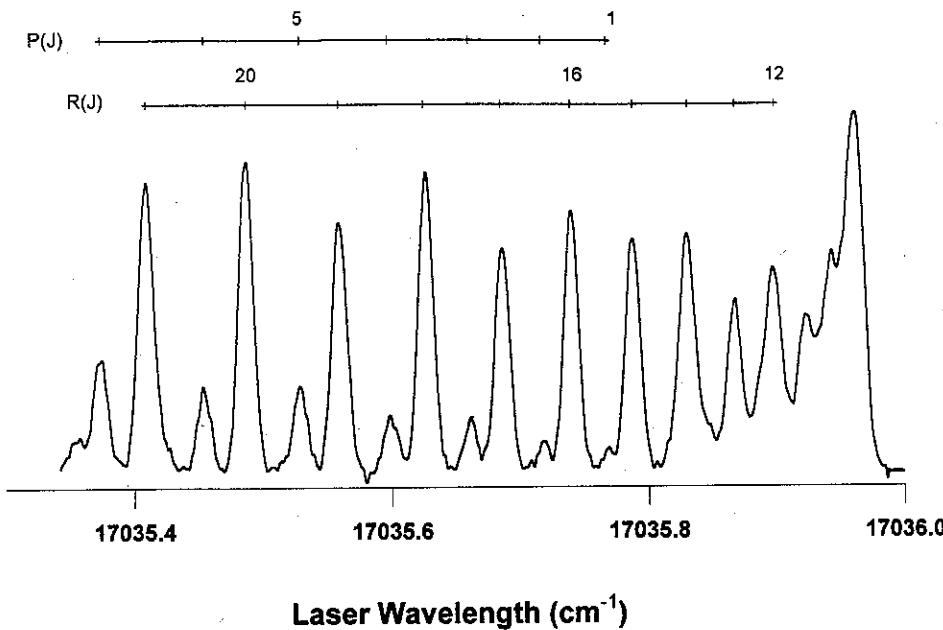


FIG. 2. Laser excitation spectrum near the head of the ($v' = 0, v'' = 4$) band. Rotational levels are assigned to the *P* and *R* branch.

The bismuth dimer is the heaviest stable diatom. The Franck-Condon factors for the $v'' = 2-5$ and $v' = 0-4$ have been reported in this paper. Answer the following questions:

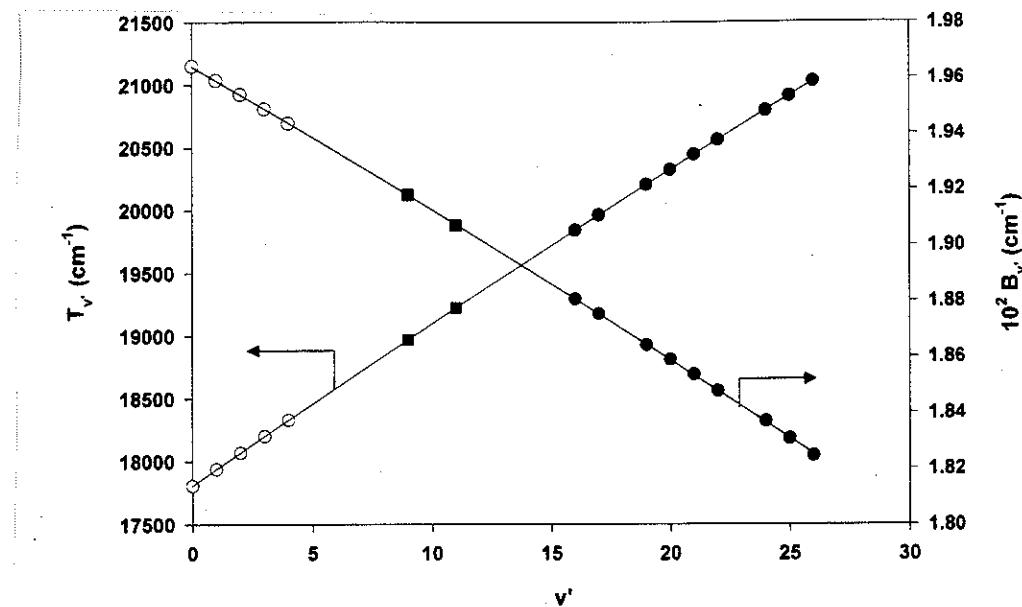
Assign all the quantum numbers of the upper state and the lower state for the transition corresponding to the first tall peak on the left side of the spectrum.

UPPER STATE

LOWER STATE

Write down an equation that would be used in the analysis of the peak frequencies in order to obtain the spectroscopic constants. You may use a running number $m = (J+1)$ for one branch and $-J$ for the other branch.

The vibrational and the rotational term values for the upper electronic state are plotted below, using data from the present paper and previous work.



Write down an equation that describes the vibrational term values in terms of the quantum number v' .

Write down an equation that describes the rotational term values in terms of the vibrational quantum number v' .

What spectroscopic constants can be obtained from the plot above? (First derive the equation for the plot, then define what intercept =, slope =).

The intensity observed in emission is related to the Franck Condon factors. As quoted from this 1999 paper,

$$I_{v,w} = \left(\frac{64\pi^4}{3} \right) c \nu_{vw} q_{vw} |R_e|^2 \left(\frac{S_J}{2J+1} \right) D(\nu_{vw}) N_v \quad [2]$$

v = vibrational level of emitting state, $A(0_u^+)$

w = vibrational level of ground state, $X^1\Sigma_g^+$

$I_{v,w}$ = observed LIF intensity for $v \rightarrow w$ emission band

$q_{v,w} = |\langle v|w \rangle|^2$ = Franck-Condon factor for $v \rightarrow w$ emission band

ν_{vw} = frequency for $v \rightarrow w$ emission band

$|R_e|^2$ = electronic transition dipole moment

S_J = rotational linestrength

$D(\nu_{vw})$ = system spectral response (detectivity) as a function of emission frequency

N_v = concentration of emitting state, v

There is an error in this formula. What is it?

Give the formula for the rotational line strength S_J .

Give the formula for the concentration of emitting state, N_v .

Suppose you were asked to obtain $q_{v,w}$ from first principles, how would you do it?

Show how $|R_e|^2$ is obtained theoretically.

The paper states that Ehret and Gerber have measured $|R_e|^2 = (1.4 \pm 0.4 \text{ Debye})^2$, independent of internuclear separation. **How could such a measurement be made?**

To obtain the experimental Franck Condon factors, the usual calibration procedures and forced normalization are carried out:

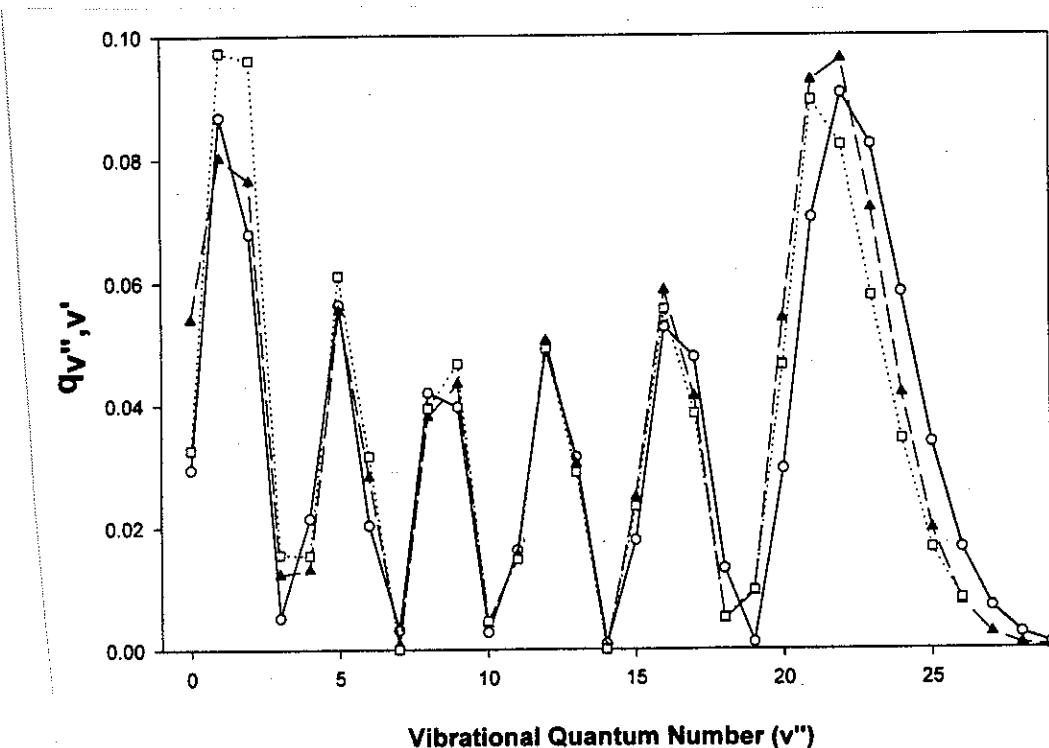
$$\frac{q_{vw}}{q_{vu}} = \left(\frac{\nu_{vw}}{\nu_{vu}} \right)^4 \left(\frac{D(\nu_{vw})}{D(\nu_{vu})} \right) \left(\frac{I_{vw}}{I_{vu}} \right). \quad [3]$$

The normalization

$$\sum_w q_{vw} = 1 \quad [4]$$

These *experimental* values are shown as ($\cdots\Box\cdots$) in the figure below.

The Franck Condon factors were also *computed from the potential energy surfaces* using the spectroscopic constants for the upper A state and the lower X state; from the spectroscopic constants reported in this 1999 paper are shown as ($--\blacktriangle--$) and from slightly different set of reported spectroscopic constants are shown as ($--\circ--$).



Explain how potential energy surfaces may be computed from spectroscopic constants.

Explain how Franck Condon factors may be computed from potential energy surfaces.

For which vibrational quantum number v' in the upper state are the Franck Condon factors that are plotted in the above figure? Explain the variation of $q_{v''v'}$ versus v'' in the figure by using schematic drawings of the two surfaces and the transitions corresponding to the plotted FC factors.

2. The visible transitions of the AuCl molecule were observed at high resolution for the first time [*J. Mol. Spectrosc.* 194, 124-7 (1999)]. The excited AuCl molecules were produced in a microwave discharge and the spectrum was recorded with a Fourier Transform emission spectrometer. A portion of the $B(\Omega=0^+) \rightarrow X^1\Sigma^+$ spectrum is shown in the figure below.

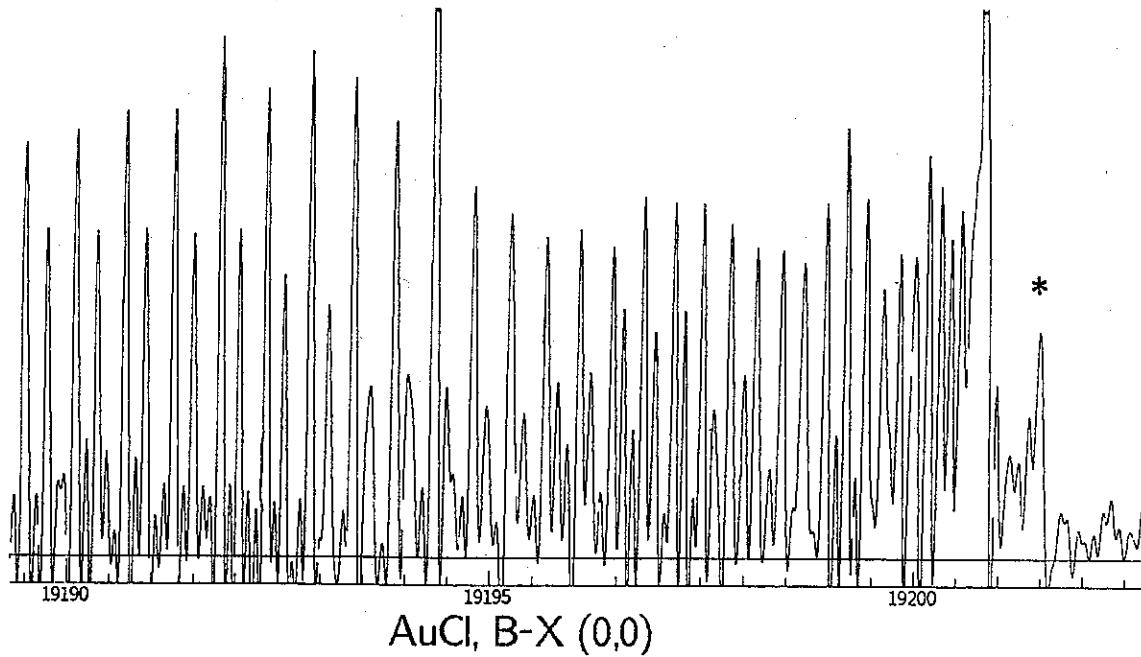


FIG. 1. A portion of the $B\ \Omega=0^+ - X^2\Sigma^+$ spectrum of Au^{35}Cl . The Au^{37}Cl bandhead is marked with an asterisk.

Explain the intensity patterns of the peaks observed.

The spectroscopic constants for the $X^1\Sigma^+$ state and the $B(\Omega=0^+)$ state are shown in the table below, all values are in cm^{-1} .

	$X^1\Sigma^+$	$B(\Omega=0^+)$
T_0	0.0	19199.2549(14)
T_1	380.7018(16)	19512.8711(15)
B_0	0.1171165(64)	0.1089616(64)
B_1	0.1165734(66)	0.1082568(64)
D_0	$0.4419(75) \times 10^{-7}$	$0.5424(73) \times 10^{-7}$
D_1	$0.4430(77) \times 10^{-7}$	$0.5542(74) \times 10^{-7}$

From an earlier observation of the 43 vibrational bands of this electronic transition, the value of $\omega_e x_e$ for $X^1\Sigma^+$ and $B(\Omega=0^+)$ states are known to be respectively, 1.30 cm^{-1} and 1.45 cm^{-1} . [Ferguson, Phys. Rev. 31, 969-72 (1928)]

Calculate ω_e and r_e for the ground state. (For comparison, relativistic density functional calculations by O. D. Häberlen and N. Rösch, *Chem. Phys. Lett.* 199, 491-6 (1992) give values that are 99.66% and 99.18% of the values reported in the 1999 paper.)

Calculate T_e for the $B(\Omega=0^+)$ state.

3. The emission spectra of the AsI molecule radical were measured in the near-infrared spectral region with a Fourier transform spectrometer [*J. Mol. Spectrosc.* 194, 250-255 (1999)]. The most prominent features in the spectrum are the six band sequences in the range 800-900 nm of the transition $b^1\Sigma^+ (b0^+) \rightarrow X^3\Sigma^- (X_10^+)$, shown in the figures below:

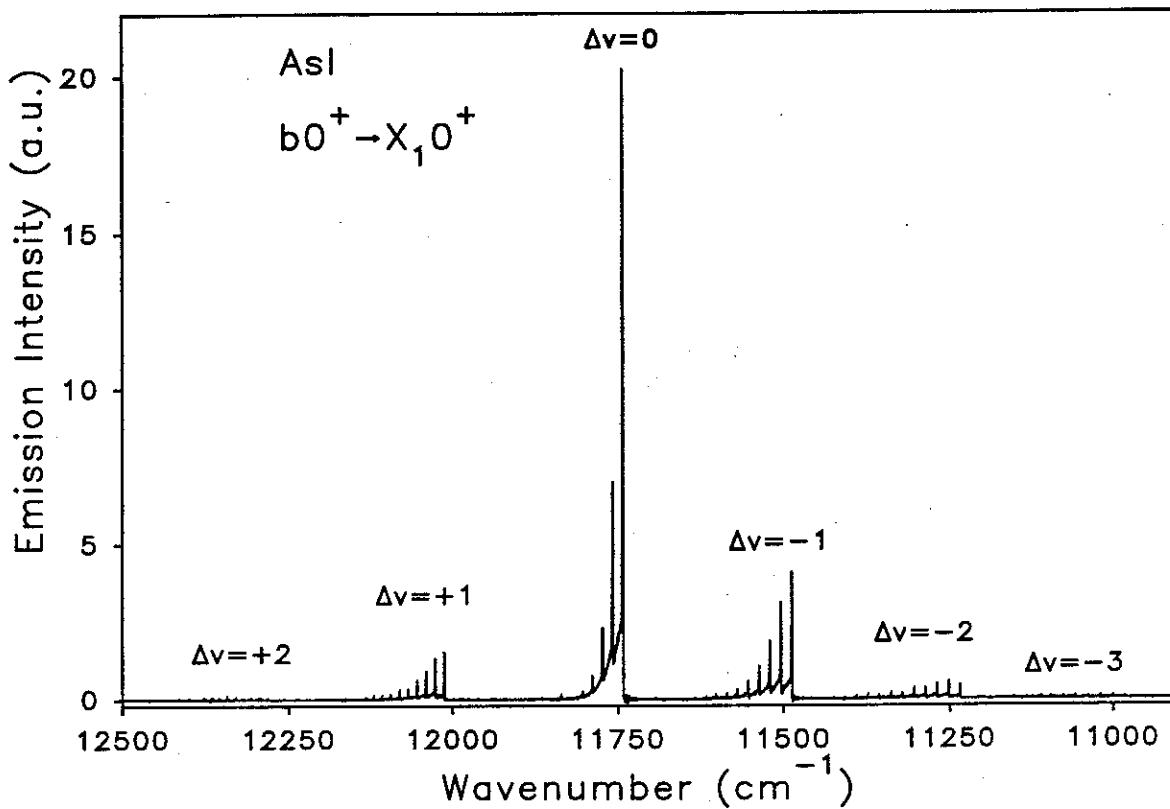


FIG. 1. Survey spectrum of the $b0^+ \rightarrow X_10^+$ transition of AsI at a spectral resolution of 0.5 cm^{-1} .

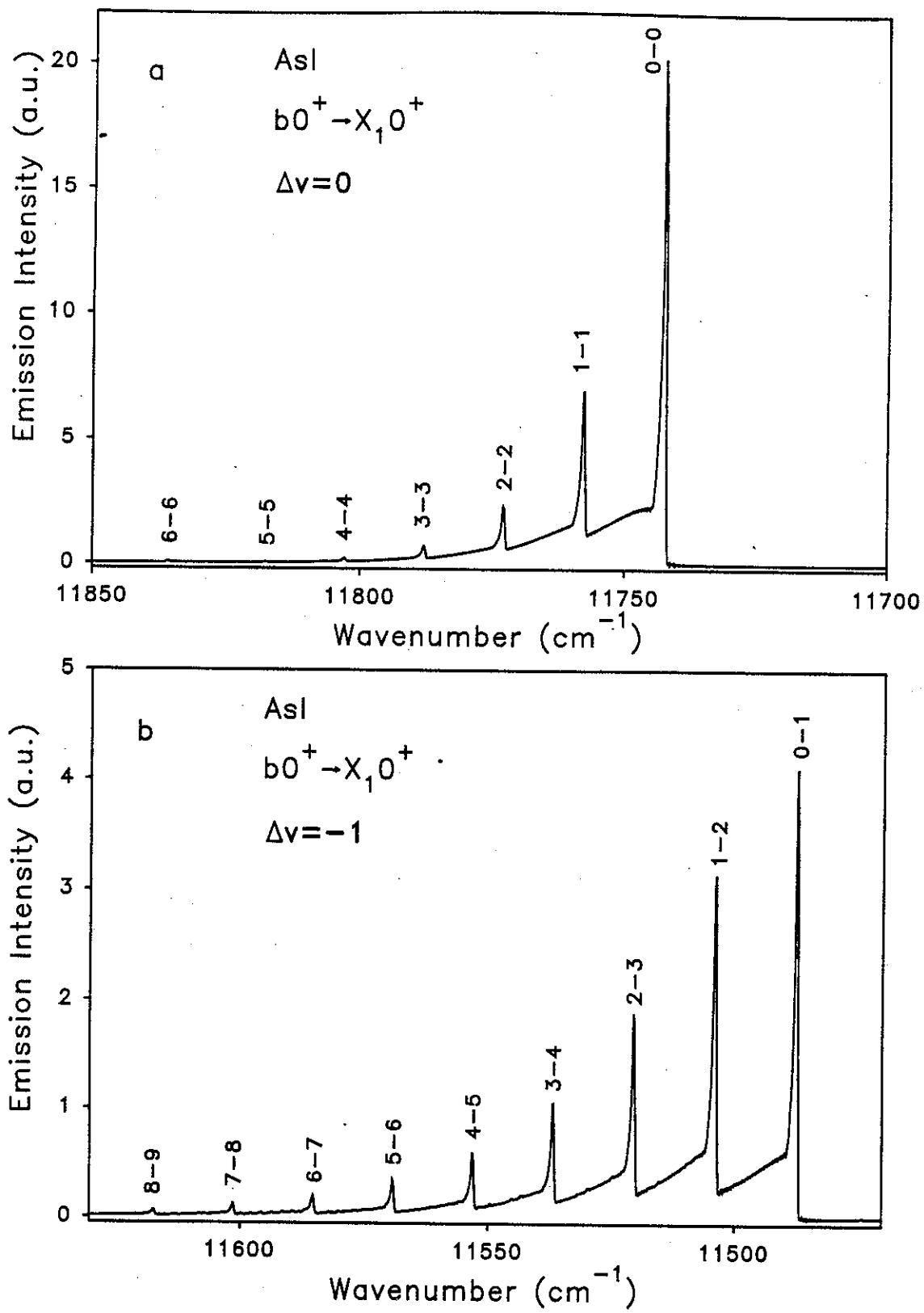


FIG. 2. Spectra of the $\Delta\nu = 0$ and $\Delta\nu = -1$ sequences of the $b0^+ \rightarrow X_10^+$ system of AsI with assignments of the *P*-branch maxima. The cm^{-1} .

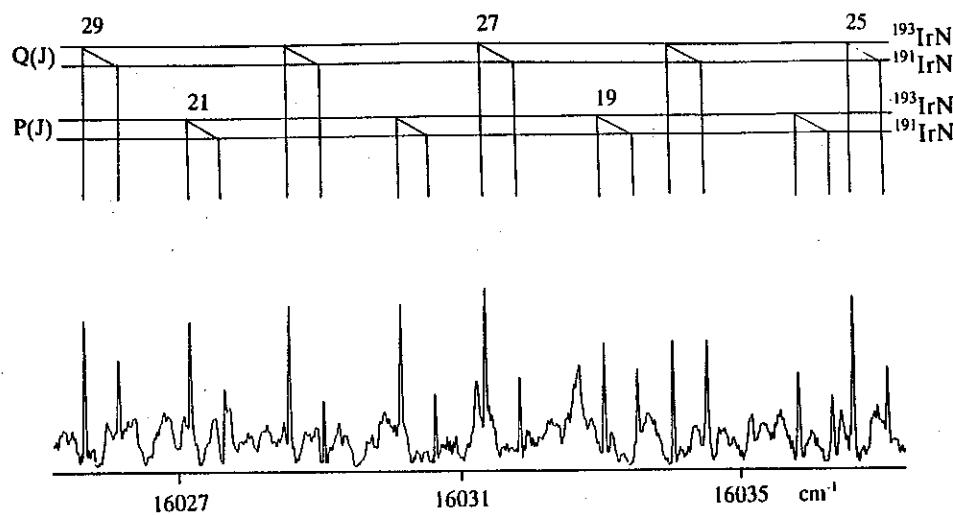
Draw a schematic of the potential surfaces and draw the transitions in Fig. 2a in the diagram. Be sure your drawing is consistent with everything in the figures on the previous page (intensity patterns, frequency differences, narrow P branches with sharp heads and broad R branches).

The identification of the transitions and the assignment of the bands observed in this paper is unambiguous. Therefore the spectroscopic constants derived represent a first complete and reliable set of molecular constants for the low-lying states of AsI. *From the spectra shown in the figures, which spectroscopic constants for which states would have been obtained and reported by the authors?*

Identify the atomic states that are the dissociation limits of AsI molecule radical.

Draw a schematic molecular orbital diagram for this molecule, identifying the As and I orbitals involved in the bonding.

4. Transition metal atoms have relatively high abundances in many stars and several transition metal hydrides and oxides have been detected. There is a strong possibility that metal nitrides can also be found, but without precise spectroscopic data, a meaningful search for these in the complex stellar spectra can not be made. The high resolution emission spectrum of IrN has been recently investigated in the 10,000-20,000 cm⁻¹ region [*J. Mol. Spectrosc.* 193, 363-375 (1999)]. In spite of overlapping with strong N₂ bands present in the same region, the IrN rotational lines were easily distinguished because of their narrow linewidths and the relatively small separation between consecutive rotational lines in a branch. see for example the expanded portion of the 1- 0 band of the $A^1\Pi \rightarrow X^1\Sigma^+$ system of IrN.



An expanded portion of the 1-0 band of the $A^1\Pi-X^1\Sigma^+$ system of IrN with the lines of the ^{191}IrN and ^{193}IrN isotopomers marked.

Why is there a Q branch?

Why the Q branch overlapping the regions of the P branch? Which has the larger rotational constant, the $A^1\Pi$ or the $X^1\Sigma^+$ state? Explain.

The molecular constants were determined by fitting the observed line positions with the customary energy level expressions for $^1\Sigma^+$ and $^1\Pi$ states. Derive these expressions for $F_v(J)$ in the $^1\Sigma^+$ case and add the required term for the $^1\Pi$ case.

The ground state $X^1\Sigma^+$ of IrN arises from the electron configuration $(1\sigma)^2 (2\sigma)^2 (1\pi)^4 (3\sigma)^2 (1\delta)^4$, whereas the $A^1\Pi$ state arises from the configuration $(1\sigma)^2 (2\sigma)^2 (1\pi)^4 (3\sigma)^2 (1\delta)^3 (2\pi)^1$, where only the atomic valence shells have been included.
Identify which of these molecular orbitals are the bonding, nonbonding, and antibonding ones.

5. Ab initio configuration interaction calculations have resulted in the theoretically calculated potential energy curves of CuN [J. Mol. Spectrosc. 194, 8-16 (1999)] shown below.

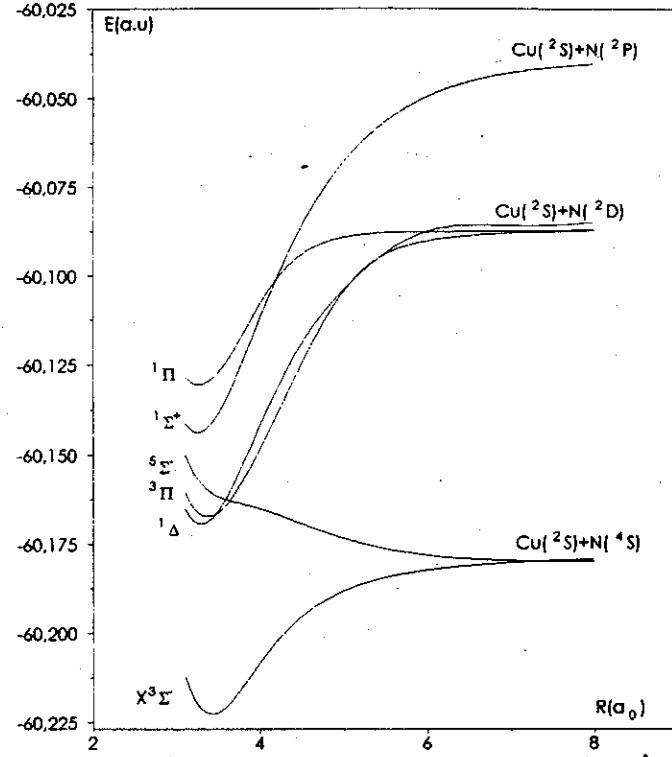


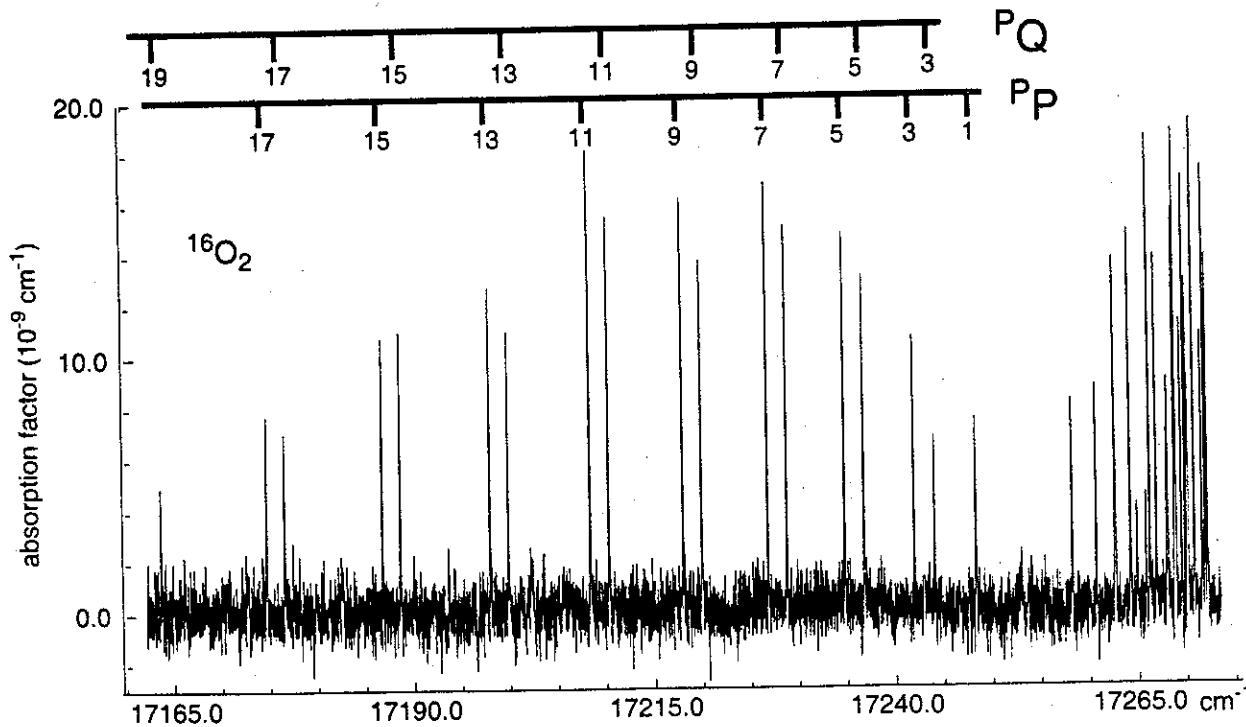
FIG. 2. CIPSI potential energy curves for the ground and excited states of CuN.

Starting from the electronic configurations of the atoms, derive the atomic states of Cu and N to which the CuN molecules dissociate.

Starting from the atomic states of the separated atoms, derive the molecular states of the CuN molecule that can result from these combinations.

Starting from the ground molecular electronic configuration, $(1\sigma)^2 (2\sigma)^2 (1\pi)^4 (1\delta)^4 (2\pi)^2 (3\sigma)^1 (4\sigma)^1$, where only the atomic valence shells have been included, that is, N: $(1s)^2$ and Cu: $(1s)^2 (2p)^6 (3s)^2 (3p)^6$ are not included, derive the molecular electronic states that can result.

6. The $b\ ^1\Sigma_g^+ \rightarrow X\ ^3\Sigma_g^-$ red atmospheric system of the O₂ molecule is, despite its small oscillator strength, of importance for light scattering and extinction studies of the Earth's atmosphere. For this reason this band system has been the subject of many studies, the latest of which was the high resolution study of reported in [J. Mol. Spectrosc. 193, 442-445 (1999)]. The (3,0) bands of this system for the ¹⁶O₂ and the ¹⁸O₂ molecules are shown below.



The molecular constants of the $X\ ^3\Sigma_g^- v = 0$ ground state are very accurately known from previous work, these can be kept constant in the analysis to find the molecular constants for the $b\ ^1\Sigma_g^+$ state. **Why is the oscillator strength so small for this band system?**

Explain the meaning of the labels $^P P$ and $^P Q$ in the P branches of $^{16}O_2$ and $^R R$ and $^R Q$ in the R branches of $^{16}O_2$ spectra shown.

Provide the quantum numbers of the upper and lower state of the first marked line (19) on the left side of the top spectrum

UPPER STATE

LOWER STATE

7. The cyanogen (NCCN) molecule is an important constituent of flames. For this reason this molecule has been the subject of many studies, the latest of which was the high resolution infrared spectra of various isotopomers of cyanogen reported in [J. Mol. Spectrosc. 193, 183-194 (1999)]. The table summarizing the nuclear spin statistical weights in this paper is reproduced below. Parity e is (+) and f is (-)

TABLE 1
Relative Nuclear Spin Statistical Weights for Cyanogen^a

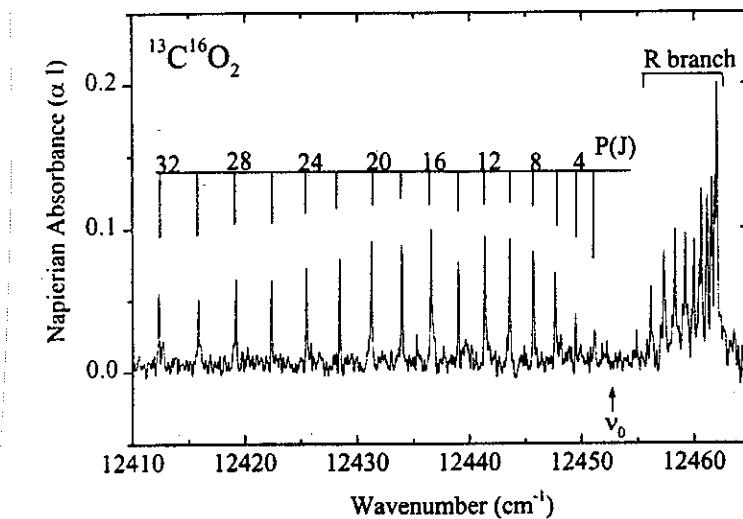
State	Parity	J	$^{12}\text{C}_2\text{ }^{14}\text{N}_2$	$^{12}\text{C}_2\text{ }^{15}\text{N}_2$	$^{13}\text{C}_2\text{ }^{15}\text{N}_2$	$^{13}\text{C}_2\text{ }^{14}\text{N}_2$
g	e	Even	2	1	5	5
		Odd	1	3	3	7
f	e	Even	1	3	3	7
		Odd	2	1	5	5
u	e	Even	1	3	3	7
		Odd	2	1	5	5
	f	Even	2	1	5	5
		Odd	1	3	3	7

^a For the non-centrosymmetric isotopomers there is no $g-u$ distinction and all rovibrational levels have the same nuclear spin statistical weight.

Derive the entries for the $^{13}\text{C}_2\text{ }^{15}\text{N}_2$ isotopomer in this table, defining all the symmetry terms used.

Given that the equilibrium bond length of the N-C bond is 1.157Å and that for the C-C bond is 1.380Å , calculate the moments of inertia of the various isotopomers given in the table.

8. The $(1\ 0^0\ 5)_{1\text{st}\ \text{or}\ 2\text{nd}} \rightarrow (0\ 0^0\ 0)$ band of $^{13}\text{C}^{16}\text{O}_2$ molecule has been reported in [J. Mol. Spectrosc. 193, 204-212 (1999)]. The figure below, taken from this paper shows the intracavity laser spectrum taken of 200 Torr of gas in the cell under conditions which gives an equivalent path length of 13.5 km.



The 1st or 2nd (subscript 1 or 2) notation refer to the first or second member of the Fermi dyad that result from coupling of the nearly degenerate $(1\ 0^0\ 5)$ and the $(0\ 2^0\ 5)$ vibrational states. The Fermi dyad resulting from the near degeneracy of $(1\ 0^0\ 0)$ and $(0\ 2^0\ 0)$ of $^{12}\text{C}^{16}\text{O}_2$ is the most well known case. ***Explain the notation used for the upper combination state in the figure.***

Derive the expression for G_v in this case.

Given the experimental numerical values in cm^{-1} in Table 7 below,

show how to determine G_v , B_v , D_v .

9. The high resolution infrared study of the ν_{14} , ν_{17} , and ν_{18} of $^{11}\text{B}_2\text{H}_6$ and $^{10}\text{B}^{11}\text{BH}_6$ molecules has been reported in [J. Mol. Spectrosc. 191, 331-342 (1998)]. The figure below, taken from this paper shows the diborane molecule and the symmetry species of the normal modes.

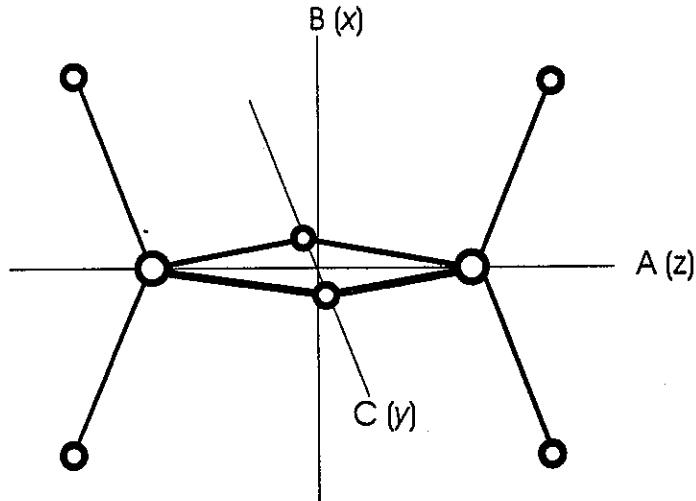


FIG. 1. Model of the diborane molecule showing the position of the atoms and the inertial axes.

TABLE 1
 D_{2h} Point Group for $^{11}\text{B}_2\text{H}_6$ and $^{10}\text{B}_2\text{H}_6$

	I	C_2^z	C_2^x	C_2^y	i	σ_{xy}	σ_{yz}	σ_{xz}	
A_g	1	1	1	1	1	1	1	1	$\nu_1, \nu_2, \nu_3, \nu_4$
A_u	1	1	1	1	-1	-1	-1	-1	ν_5
B_{1g}	1	1	-1	-1	1	1	-1	-1	R_z, ν_{15}
B_{1u}	1	1	-1	-1	-1	-1	1	1	$T_z, \nu_{16}, \nu_{17}, \nu_{18}$
B_{2g}	1	-1	-1	1	1	-1	-1	1	R_y, ν_{11}, ν_{12}
B_{2u}	1	-1	-1	1	-1	1	1	-1	T_y, ν_{13}, ν_{14}
B_{3g}	1	-1	1	-1	1	-1	1	-1	R_x, ν_6, ν_7
B_{3u}	1	-1	1	-1	-1	1	-1	1	$T_x, \nu_8, \nu_9, \nu_{10}$

Derive these results, i.e., derive the irreducible representations of the normal vibrations of the diborane molecule.

$$E = U_\alpha(R_e) + (v+\frac{1}{2})v_e - x_e v_e (v+\frac{1}{2})^2 + y_e v_e (v+\frac{1}{2})^3 + B_e J(J+1) - D_e [J(J+1)]^2 - \alpha_e (v+\frac{1}{2})J(J+1) + Y_{00}$$

where all spectroscopic quantities are expressed in energy units (or the corresponding frequency or wavenumbers). In energy units,

$$B_e \equiv \hbar^2 / 2\mu R_e^2$$

$$hx_e v_e \equiv \frac{1}{4} B_e^2 / (hv_e)^2 \cdot \left\{ \left(\frac{10}{3}\right) B_e [U'''(R_e) R_e^3]^2 / (hv_e)^2 - U^{iv}(R_e) R_e^4 \right\}$$

$$D_e \equiv 4 B_e^3 / (hv_e)^2$$

$$\alpha_e \equiv -2 B_e^2 / hv_e \cdot \left\{ 3 + 2 B_e [U'''(R_e) R_e^3]^2 / (hv_e)^2 \right\}$$

$$Y_{00} \equiv B_e^2 / 16(hv_e)^2 \cdot \left\{ U^{iv}(R_e) R_e^4 - \left(\frac{14}{9}\right) B_e [U'''(R_e) R_e^3]^2 / (hv_e)^2 \right\}$$

$$hv_e \equiv (h/2\pi) [U''(R_e) / \mu]^{1/2}$$

Energy Conversion Factors $300 \text{ MHz} = 0.01 \text{ cm}^{-1}$, $208.5 \text{ cm}^{-1} = 300 \text{ K}$

		erg	eV	cm ⁻¹
1 erg	=	1	6.2415×10^{11}	5.0340×10^{15}
1 eV	=	1.6022×10^{-12}	1	8065.5
1 cm ⁻¹	=	1.9865×10^{-16}	1.23985×10^{-4}	1
1 kcal/mole	=	6.9478×10^{-14}	4.3364×10^{-2}	349.75
1 hartree	=	4.3598×10^{-11}	27.212	219,474.6

		kcal/mole	hartree
1 erg	=	1.4393×10^{13}	2.2937×10^{10}
1 eV	=	23.060	3.6749×10^{-2}
1 cm ⁻¹	=	2.8591×10^{-3}	4.55634×10^{-6}
1 kcal/mole	=	1	1.5936×10^{-3}
1 hartree	=	627.51	1

