Problem Set 1

Chemistry 448

Cynthia J. Jameson

1. Compute the probability of getting heads 5 times in 5 tosses of a coin.
2. Compute the probability of getting heads 3 times and tails twice in the specific sequence *hhhtt.*
3. Compute the probability of 4 heads and 2 tails in any sequence.
4. What is the probability of getting a head on the sixth toss after heads have been seen to come up five times in a row?
5. Compute the probability of throwing a four twice in two throws of a pair of dice, (i.e., a total of 4 on the two dice on each toss).
6. An urn contains 3 white and 5 black balls. Suppose one ball is drawn and color unnoted, and then a second ball is drawn. What is the probability that the second ball is white?
7. In tossing 10 coins, what is the probability of getting (a) 5 heads in just one way, (b) 5 heads in any sequence, (c) 5 or more heads?
8. In tossing 10 coins that are weighted so that the chance of getting a head in any one toss is twice as great as getting a tail, what is the probability of getting 5 heads in any sequence?
9. Five distinguishable balls are dropped at random into three distinguishable boxes. What is the probability that the first box contains 3 balls, the second box 2 balls, and the third box none?
10. Repeat the calculation of problem 9 under the conditions that the first box has two compartments, which gives it an *a priori* probability twice as great as the other two boxes.
11. Three cabinets identical in appearance each have two drawers. The first drawer contains a gold coin in each drawer, the second a silver coin in each drawer, and the third a gold coin in one drawer and a silver coin in the other. Choose one cabinet at random.
(a) What is the probability that it contains the coins of different metals?
(b) If one of the drawers is opened, and found to contain a gold coin, what is the probability that the other drawer in the same cabinet contains a silver coin?
12. An urn contains five balls labeled a, b, c, d, and e. What is the probability of drawing three balls in the exact order a, c, and e?
13. Consider the five molecules of hydroxy acid. If each monomeric molecule is tagged so as to be distinguishable, in how many ways could the five molecules dimerize, i.e., form two dimers with one monomer unreacted? In doing this problem, it must be realized that the number of possibilities depends on recognizing the order in which the monomers form each molecule, and the order in which the two dimeric molecules are formed. Calculate the number of possibilities for each possible combination of assumptions.
14. Deduce a formula for the number of ways *n* distinguishable objects may be placed in *c* numbered boxes with no restrictions on the number of objects per box. Include all possible distributions.

15. Deduce a formula for the number of ways *n* indistinguishable objects may be placed in *c* numbered boxes with no more than one object to a box, $n \leq c$.
16. Deduce a formula for the number of ways *n* indistinguishable objects may be placed in *c* numbered boxes with no restrictions on the number of objects per box.
17. Deduce a formula for the total number of sequences C_t, having *t* empty sites in sequence in all possible configurations when *n* side chains are distributed on *S* carbon atoms along the main chain of a single paraffinic molecule with not more than one side chain to a site.
18. Prove that the number of ways of drawing $m_1, m_2, m_3,$ ... objects out of a total of *n* objects is given by:

$$C_n^{m_1,m_2,m_3,\ldots} = \frac{n!}{m_1!m_2!m_3!\ldots}$$

19. Make a graph of the number of ways each possible number can come up on tossing two dice. What do you conclude from the plot?