Chemistry 448

Problem Set #1

1. The probability of getting heads 5 times in 5 tosses is

$$\left[\left(\frac{1}{2}\right)\right]^5$$

2. The probability of getting the sequence hhhtt is same as above

$$\left[\left(\frac{1}{2}\right)\right]^3\left[\frac{1}{2}\right]^2$$

(The probability of getting any one particular sequence in 5 tosses is $(\frac{1}{2})^5$)

3. Probability of 4 heads and 2 tails in any sequence

$$\underbrace{\frac{6!}{4!\,(6-4)!}}_{\text{no. of ways of arranging 4 of one kind and 2 of another kind}} * \underbrace{\left[\frac{1}{2}\right]^4\left[\frac{1}{2}\right]^2}_{\text{probability of getting any one particular sequence}}$$

4. Probability of getting a head on the sixth toss is $\frac{1}{2}$ no matter how many heads have come up previously.

5. probability of throwing a four twice in two throws of a pair of dice is

$$\left\{ \underset{\substack{\uparrow \\ \text{only 3 ways of getting} \\ \text{a sum of four} \\ 3+1,\ 1+3,\ 2+2}}{3} * \left[\frac{1}{6}\right]\left[\frac{1}{6}\right] \right\}^2 = \frac{1}{144}$$

From 3 white and 5 black balls

6. probability of first ball is white, second ball is white $= \left(\frac{3}{8}\right)\left(\frac{2}{7}\right)$

Probability of first ball is black, second ball is white $= \left(\frac{5}{8}\right)\left(\frac{3}{7}\right)$

$\therefore$ Probability of second ball is white, regardless of the color of the first ball =

$$\left(\frac{3}{8}\right)\left(\frac{2}{7}\right) + \left(\frac{5}{8}\right)\left(\frac{3}{7}\right)$$

7. Tossing 10 coins

a) probability of getting 5 heads in any one way or outcome $= \left[\frac{1}{2}\right]^5 \left[\frac{1}{2}\right]^5$

b) probability of getting 5 heads in any sequence =

$$\underbrace{\frac{10!}{5!\,(10-5)!}}_{\text{no. of ways of getting 5 of one kind 5 of the other}} \left[\frac{1}{2}\right]^5 \left[\frac{1}{2}\right]^5$$

c) probability of getting 5 or more heads =

$$\left[\frac{10!}{5!\,(10-5)!} + \frac{10!}{6!\,(10-6)!} + \frac{10!}{7!\,(10-7)!} + \frac{10!}{8!\,(10-8)!} + \frac{10!}{9!\,(10-9)!} + 1\right] * \left[\frac{1}{2}\right]^{10}$$

8. If chance of getting a head in any one toss is twice as great as getting a tail,

Probability of getting 5 heads in any sequence is (compare with problem 7 b)

$$\frac{10!}{5!(10-5)!}\left[\frac{2}{3}\right]^5\left[\frac{1}{3}\right]^5$$

probability of getting a head in one toss $= \frac{2}{3}$

9. Probability of each one outcome for 5 balls is

$$\left[\frac{1}{3}\right]^5$$

no. of ways of ~~getting~~ the distribution 3, 2, 0 in ~~this~~ particular order is

$$\frac{5!}{3!\,2!\,0!}$$

Answer: The probability that box number 1 contains 3 balls, box number 2 contains 2 balls and box number 3 contains 0 ball =

$$\frac{5!}{3!\,2!\,0!}\left[\frac{1}{3}\right]^3\left[\frac{1}{3}\right]^2\left[\frac{1}{3}\right]^0 = 0.411523$$

10. Now instead of probabilities being $\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$ for boxes 1, 2, and 3 respectively, they are $\frac{1}{2}, \frac{1}{4}, \frac{1}{4}$ respectively

$$P = \frac{5!}{3!\,2!\,0!}\left(\frac{1}{2}\right)^3\left(\frac{1}{4}\right)^2\left(\frac{1}{4}\right)^0 = 0.078125$$

which is larger than 0.0411523.

Another way of getting this answer is by thinking of four boxes:

Box A	Box B	Box C	Box D
0	3	2	0
1	2	2	0
2	1	2	0
3	0	2	0

with $\frac{1}{4}$ as intrinsic probability of being in one of the four boxes

$$P = \left\{\frac{5!}{0!\,3!\,2!\,0!} + \frac{5!}{1!\,2!\,2!\,0!} + \frac{5!}{2!\,1!\,2!\,0!} + \frac{5!}{3!\,0!\,2!\,0!}\right\}\left(\frac{1}{4}\right)^5$$

$$= 0.078125$$

11. 3 choices are GG SS GS

a) probability that the cabinet contains coins of different metals $= \frac{1}{3}$

b) if one drawer is opened and found to contain a gold coin, the 2 choices are GG or GS

so the probability that the other drawer in the same cabinet contains a silver coin is $\frac{1}{2}$

12. The probability of drawing 3 balls in the exact order a card is

$\left(\frac{1}{5}\right)\left(\frac{1}{4}\right)\left(\frac{1}{3}\right)$ — one out of 3 choices

one out of 4 choices

one out of 5 choices

13. Five molecules A B C D E

a) Assume that the order in which the monomers form a dimer is recognizable (that is AB is different from BA)

and the order in which the dimers are formed is also noted, that is AB CD is different from CD AB

There are 5 "positions" possible

1 first monomer in dimer I
2 second monomer in dimer I
3 first monomer in dimer II
4 second monomer in dimer II
5 unchanged monomer

13 continued

number of ways $= 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5!$

b) Assume that the order in which the monomers form a dimer is immaterial and the order in which the dimers are formed is not noted. There are only the 3 following ways of classifying A B C D and E:

in dimer I
in dimer II
or unchanged monomer

Answer = no. of ways of having the distribution 2, 2, 1 in <u>any</u> order

$$= \frac{5!}{2!\,2!\,1!}$$

c) Assume that the order in which the monomers form a dimer is recognizable (that is AB and BA are different dimers) but the order in which the dimers are formed is not noted, that is AB and CD is not different from CD and AB.

The answer is the same as a) <u>except</u> divide by 2! which is the number of ways of arranging dimer I and II in order

Answer $= \dfrac{5!}{2!}$

13 continued

d) Assume that the order in which monomers form a dimer is not recognizable but the order in which the dimers are formed is noted.

Answer is the same as b) except multiply by 2! which is the number of ways of arranging dimers I and II

$$\frac{5!}{2!\,1!}$$

or else the same as a) except divide by 2! which is the number of ways of forming a dimer from 2 monomers

$$\frac{5!}{2!}$$

14. The number of ways of placing n distinguishable objects in c numbered boxes with no restrictions on the number per box, including all possible distributions =

$$(c)(c)(c)\cdots(c) = c^n$$

c choices for each object

15. The number of ways in which n INDISTINGuishable objects may be placed in c numbered boxes with no more than one object to a box and $n \le c$ =

$$\frac{c!}{n!\,(c-n)!}$$

since n! = no. of ways of permuting the filled boxes and (c−n)! = no. of ways of

permuting the unfilled boxes.
That is, classify the boxes into two kinds filled and empty, the number of ways of arranging c boxes into n filled and $(c-n)$ empty $= \dfrac{c!}{n!\,(c-n)!}$

16. number of ways n INDISTINguishable objects may be placed in c numbered boxes with no restrictions on the number per box $= \dfrac{(n+c-1)!}{n!\,(c-1)!}$

Imagine $(c-1)$ partitions and n objects on a string. the number of ways of arranging n objects and $c-1$ partitions is $\dfrac{(n+c-1)!}{n!\,(c-1)!}$

17. s total sites, n sites are each occupied by one side chain. How many sequences have t empty sites in a row in all possible configurations?
No. of ways of having n occupieds and $(s-n)$ empties is
$$\frac{s!}{(s-n)!\,n!}$$
Of these, a fewer number have t empty sites in a row.

17 continued ...

Consider t empty sites in a row preceding an occupied site (this guarantees that there are no more than t empties in a row).

The number of remaining sites is

$$s-\underbrace{(t+1)}$$

t empties followed by 1 occupied

$(n-1)$ = the number of occupieds to be permuted among the $s-(t+1)$ sites

$(s-n-t)$ = the number of empties to be permuted among the $s-(t+1)$ sites.

The number of ways of doing this is

$$\frac{(s-(t+1))!}{(s-n-t)!\,(n-1)!}$$

But there are n occupied sites that the t empty sites can precede, plus one should add one for when the t empty sites are at the very end (no need to be followed by an occupied site)

Therefore, multiply the above number by $(n+1)$

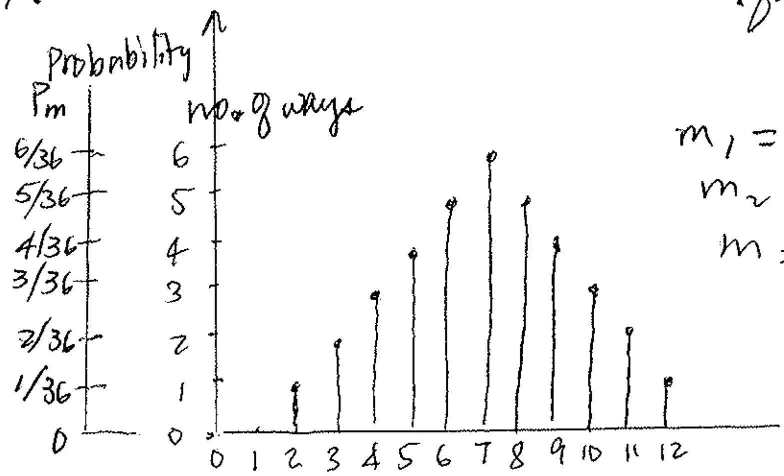
Answer $= \dfrac{(n+1)\,[s-(t+1)]!}{(s-n-t)!\,(n-1)!}$ number of sequences having t empty sites in a row in all possible configurations

18. The number of ways of drawing m_1, m_2, m_3 objects out of a total of n objects =

$$\frac{n!}{m_1!\, m_2!\, m_3! \cdots}$$

since the first m_1 objects can be picked in $m_1!$ ways, the next m_2 objects in $m_2!$ ways and all n objects can be arranged in $n!$ ways.

19. See lecture notes for the answer.



$$\left.\begin{array}{l} m_1 = 1, 2, 3, \cdots 6 \\ m_2 = 1, 2, 3, \cdots 6 \end{array}\right\} \text{each die}$$

$$m = m_1 + m_2$$

The number m that corresponds to the maximum number of configurations is 7, is the most probable.

The distribution is symmetric around $m = 7$ so 7 should also be the average number

$$\langle m \rangle = \sum_{m=2}^{12} P_m \cdot m$$