Problem Set 7
Chemistry 448
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The relation between the magnetic moment $\boldsymbol{\mu}$ (a vector) of a particle and its intrinsic angular momentum $\boldsymbol{I}$ (also a vector) is given by a fundamental property of the particle, the magnetogyric ratio γ.

$$\boldsymbol{\mu} = \gamma \, \boldsymbol{I}$$

where components of $\boldsymbol{I}$ are $I\hbar$, $(I\text{-}1)\hbar$, $(I\text{-}2)\hbar$, ... $-I\hbar$.
For example, $\gamma_p = 2.675\ 222\ 099 \times 10^8$ rad $s^{-1}T^{-1}$ for the proton. γ is written as

$$\gamma = g\mu_N/\hbar.$$

Here μ_N the nuclear magneton, is $e\hbar/2m_p$ in which the mass of the proton is m_p . Expressed in this way in terms of the fundamental constants, the g value for the particle then carries the intrinsic property of the particle relating $\boldsymbol{\mu}$ and $\boldsymbol{I}$.
For the electron $\gamma_e = -1.760859770 \times 10^{11}$ rad $s^{-1}T^{-1}$.

$$\gamma_e = -\, g_e\mu_B/\hbar.$$

Here μ_B the Bohr magneton, is $e\hbar/2m_e$ and $g_e = 2.0023193043617$.
The Bohr magneton is 9.274×10^{-24} Joule Tesla^{-1} or 9.274×10^{-21} erg gauss^{-1}.

The potential energy of a particle with magnetic moment $\boldsymbol{\mu}$ in a magnetic field $\boldsymbol{B}$ is given by

$$E_{mag} = -\, \boldsymbol{\mu} \bullet \boldsymbol{B}$$

1. Consider a collection of N independent atoms, each having an unpaired electron in a ^2S state. Suppose that this system is placed in a magnetic field of magnitude B. Draw a diagram showing the effect of the applied field on the ground state energy level. Derive an expression for the number of atoms in each of the levels shown in your diagram.

2. The magnetic susceptibility χ of a substance is the induced magnetic moment per unit field strength. Derive a general expression for the magnetic susceptibility of the system described above in terms of the Bohr magneton μ_B, and the absolute temperature T.

3. Show how this general expression can be simplified for low fields and/or high temperatures. show that this limiting form is an excellent approximation b even at room temperature and with a magnetic field of 10^4 gauss = 1 Tesla. Boltzmann's constant k = $1.3806503 \times 10^{-16}$ erg K^{-1} molecule^{-1}, or $1.3806503 \times 10^{-23}$ Joule K^{-1} molecule^{-1}.

4. Describe qualitatively what happens to the susceptibility χ as one goes to very low temperature and/or very high fields. Plot χ as a function of B/T where T is the absolute temperature.

5. The molecule O_2 has a $^3\Sigma$ ground state. Considering only this triplet ground state, derive an expression for the magnetic susceptibility of the O_2 molecule in terms of the Bohr magneton and the temperature.

6. A number of organic molecules have singlet ground states but also have a low-lying triplet excited state. Taking this separation between the singlet ground state and the triplet excited state to be E_T, sketch these low-lying states in the absence and in the presence of a magnetic field. Derive an expression for the magnetic susceptibility of such a molecule, assuming $\mu_B B \ll kT$.

7. Show that if $kT \gg E_T$, the magnetic susceptibility will be nearly temperature independent.