Nuclear magnetic shielding and chirality IV. The odd and even character of the shielding response to a chiral potential

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We investigate the odd and even character of the shielding response in a chiral molecule (modeled by a Ne_8 helix) when subjected to a chiral potential. We establish that the diastereomeric splittings are a measure of odd powers of V_{odd}. Implications for diastereomeric, splittings of Xe in handed cages with handed tethers are discussed. © *2004 American Institute of Physics.*
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I. INTRODUCTION

Recently we investigated the shielding tensor of Xe and naked spins embedded in Ne helices.[1–3] In order to manifest the chiral aspects of these systems, we further embedded them in helices of partial charges. These Xe complexes are meant to model the diastereomeric aspects of Xe in cryptophane cages with attached chiral tethers.[4,5] The splitting of the Xe resonances is a manifestation of the diastereomerism. In order to investigate the phenomenon of ''induced chirality,'' we replaced Xe by a naked spin. In reality, one could imagine replacing the Xe with a ^{3}He atom.

Experimentally, only the isotropic ^{129}Xe chemical shifts of the Xe@tethered-cryptophane cages were measured.[4,5] Unlike our earlier work where the full tensors were discussed, in this paper we shall only discuss the isotropic chemical shifts relative to an isolated Xe atom, which, being scalars, are more readily analyzed. The quantum mechanical calculations provide the full tensors, of course. In this paper, we investigate the role of the partial charge helices in greater depth. We vary both the sign and the magnitude of the charges. In addition, we examine the chirality-induced (or not induced) by adding or subtracting the potential due to partial charge helices of the same or opposite charges and handedness. Of course, in reality we cannot physically superimpose opposite handed tethers on a given molecule. We emphasize that this superposition is not the same as inserting more chiral links to the tether.

II. THE POTENTIALS

The Ne helices and the Xe electrons feel a potential due to the partial charges. In the naked spin-Ne helix complex, only the Ne electrons interact with the helix of partial charges. Thus, the potential per electron at point $\mathbf{r}$ is the same for both systems and our analysis begins with a potential at point $\mathbf{r}$ due to an outer helix of partial charges of value $\pm q$. Namely, the potential, $V(\pm q,\mathbf{r})$ may be written as

$$V(\pm q,\mathbf{r}) = \pm q \Sigma_{i=1}^{N} (-e)/|\mathbf{R}_i - \mathbf{r}|, \tag{1}$$

where $\mathbf{R}_i$ is the position of the ith partial charge. Clearly, we may define a potential per partial charge as, $V(\mathbf{r})$:

$$V(\pm q,\mathbf{r}) = \pm q V(\mathbf{r}). \tag{2}$$

When $V(\mathbf{r})$ arises from a right-handed helix of partial charges, we may write $V(\mathbf{r})$ as $[V(\mathbf{r})]_r$ and express it as

$$[V(\mathbf{r})]_r \equiv V_e(\mathbf{r}) + V_o(\mathbf{r}), \tag{3}$$

where

$$V_e(-\mathbf{r}) = V_e(\mathbf{r}) \tag{4a}$$

and

$$V_o(-\mathbf{r}) = -V_o(\mathbf{r}). \tag{4b}$$

Since, to within a rotation

$$[V(\mathbf{r})]_\ell = [V(-\mathbf{r})]_r \tag{5}$$

we have for the left-handed helix

$$[V(\mathbf{r})]_\ell = V_e(\mathbf{r}) - V_o(\mathbf{r}). \tag{6}$$

It is important to realize that achiral potentials may also be written as the sums of even and odd potentials under parity. However, in that case, there is always a rotation, $\mathcal{R}$, such that $\mathcal{R}V(-\mathbf{r})\mathcal{R}^{-1} = V(\mathbf{r})$.

Explicitly, we may expand $V_e(\mathbf{r})$ and $V_o(\mathbf{r})$ in spherical harmonics,

$$V_e(\mathbf{r}) = \Sigma_{l=0,2,\ldots} \Sigma_m R^e_{lm}(r) Y_{lm}(\theta,\phi) \tag{7a}$$

and

$$V_o(\mathbf{r}) = \Sigma_{l=1,3,\ldots} \Sigma_m R^o_{lm}(r) Y_{lm}(\theta,\phi). \tag{7b}$$

Because the partial charge helices are symmetric about $\mathbf{r}=0$, they do not possess a dipole, that is

$$V_o(\mathbf{r}) \sim \Sigma_m R^o_{3m}(r) Y_{3m}(\theta,\phi) + \text{higher order}. \tag{8}$$

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As an aside, we note that xyz, the odd potential used in Condon and Eyring's original discussion of the optical activity of an anharmonic oscillator,[6] is a sum of Y_{3m} spherical harmonics.

III. EXPANSIONS IN q

We now come to the capital point of this paper. Although the actual calculations of nuclear magnetic shieldings carried out in this work are finite field calculations, we assume that q is small enough that a power series in q is valid. We assume the series expansion is valid for both $\pm q$ and $[V(\mathbf{r})]_\ell$ and $[V(\mathbf{r})]_r$. Thus, we may obtain almost all the expansion coefficients by linear combinations of $\pm q$ and ℓ and r potentials.

There are four unique nuclear magnetic shieldings: $\sigma_{\mathcal{R}r}(+q)$, $\sigma_{\mathcal{R}r}(-q)$, $\sigma_{\mathcal{R}\ell}(+q)$ and $\sigma_{\mathcal{R}\ell}(-q)$. The diastereomeric splittings are

$$\delta(\pm q)=\sigma_{\mathcal{R}r}(\pm q)-\sigma_{\mathcal{R}\ell}(\pm q). \tag{9}$$

We now expand each of the four shieldings in a power series in q, as advertised. We presubtract the $q=0$ term. The notation will be explained after we carry out the expansions, which are

$$\sigma_{\mathcal{R}r}(\pm q)=\pm q(\sigma_e+\sigma_o)+(q^2/2!)(\sigma_{ee}+\sigma_{oo}+2\sigma_{eo})+\pm(q^3/3!)(\sigma_{eee}+\sigma_{ooo}+3\sigma_{eoo}+3\sigma_{eeo})+\ldots \tag{10}$$

and

$$\sigma_{\mathcal{R}\ell}(\pm q)=\pm q(\sigma_e-\sigma_o)+(q^2/2!)(\sigma_{ee}+\sigma_{oo}-2\sigma_{eo})+\pm(q^3/3!)(\sigma_{eee}-\sigma_{ooo}+3\sigma_{eoo}-3\sigma_{eeo})+\ldots. \tag{11}$$

The subscripts represent the order in perturbation theory that V_e and/or V_o appears. For example, σ_{oo} is second order in V_o; σ_{oee} represents V_o and V_e^2. The ordering is not explicit in the mixed terms in a multinomial. That is, σ_{oee} is not distinguished from σ_{eoe} or σ_{eeo}. The factor of 3 represents these different possibilities. The diastereomeric splittings are thus

$$\delta(\pm q)=(\pm 2q)\sigma_o+(q^2/2!)(4\sigma_{eo})\pm(q^3/3!)2(\sigma_{ooo}+3\sigma_{eeo})+\ldots. \tag{12}$$

Clearly the diastereomeric splittings are a measure of odd powers of V_o.

The mean nuclear magnetic shieldings, $[\sigma_{\mathcal{R}r}(\pm q)+\sigma_{\mathcal{R}\ell}(\pm q)]/2$ are

$$[\sigma_{\mathcal{R}r}(\pm q)+\sigma_{\mathcal{R}\ell}(\pm q)]/2=(\pm q)\sigma_e+(q^2/2!)(\sigma_{ee}+\sigma_{oo})\pm(q^3/3!)(\sigma_{eee}+3\sigma_{eoo})+\ldots. \tag{13}$$

It is readily seen that σ_e, σ_o, $(\sigma_{ee}+\sigma_{oo})$, σ_{eo}, $(\sigma_{ooo}+3\sigma_{eeo})$ and $(\sigma_{eee}+3\sigma_{eoo})$ may be obtained by analysis of linear combinations of the shieldings with the same and opposite charges.

By adding and subtracting Eqs. (10) and (11) we can find the quantities, $1/4[\sigma_{\mathcal{R}r}(+q)-\sigma_{\mathcal{R}\ell}(+q)-\sigma_{\mathcal{R}r}(-q)+\sigma_{\mathcal{R}\ell}(-q)]/q$, which when plotted against q^2 gives the intercept

$$\sigma_o=\lim_{q\to 0}1/4[\sigma_{\mathcal{R}r}(+q)-\sigma_{\mathcal{R}\ell}(+q)-\sigma_{\mathcal{R}r}(-q)+\sigma_{\mathcal{R}\ell}(-q)]/q \tag{14}$$

and the slope at the limit $q\to 0$ is $[\sigma_{ooo}+3\sigma_{eeo}]/3!$. On the other hand, $1/4[\sigma_{\mathcal{R}r}(+q)+\sigma_{\mathcal{R}\ell}(+q)-\sigma_{\mathcal{R}r}(-q)-\sigma_{\mathcal{R}\ell}(-q)]/q$ plotted against q^2 gives the intercept

$$\sigma_e=\lim_{q\to 0}1/4[\sigma_{\mathcal{R}r}(+q)+\sigma_{\mathcal{R}\ell}(+q)-\sigma_{\mathcal{R}r}(-q)-\sigma_{\mathcal{R}\ell}(-q)]/q \tag{15}$$

and the slope at the limit $q\to 0$ is $[\sigma_{eee}+3\sigma_{eoo}]/3!$. Furthermore, we can also find the quantities

$$\sigma_{eo}=\lim_{q\to 0}1/4[\sigma_{\mathcal{R}r}(+q)-\sigma_{\mathcal{R}\ell}(+q)+\sigma_{\mathcal{R}r}(-q)-\sigma_{\mathcal{R}\ell}(-q)]/q^2 \tag{16}$$

and

$$(\sigma_{ee}+\sigma_{oo})=\lim_{q\to 0}1/2[\sigma_{\mathcal{R}r}(+q)+\sigma_{\mathcal{R}\ell}(+q)+\sigma_{\mathcal{R}r}(-q)+\sigma_{\mathcal{R}\ell}(-q)]/q^2. \tag{17}$$

IV. SUM AND DIFFERENCE POTENTIALS

It is of interest to obtain the quantities σ_{ee} and σ_{oo}. As we have seen, these terms in the shielding expansion appear together. In order to compute them separately, we must superimpose partial charge helices of opposite handedness and carry out $(\pm q)$ expansions of the shieldings. From perturbation theory, we know that the resulting terms in the expansion may be directly related to the expansions in terms of more physically realistic potentials. Thus

$$\pm 2qV_o=\pm q\{[V(\mathbf{r})]_r-[V(\mathbf{r})]_\ell\}, \tag{18}$$

and

$$\pm 2qV_e=\pm q\{[V(\mathbf{r})]_r-[V(\mathbf{r})]_\ell\}. \tag{19}$$

The shieldings in these potentials may also be expanded in a power series in q, yielding

$$\sigma(\pm 2qV_o)=\pm 2q\sigma_o+[(2q)^2/2!]\sigma_{oo}\pm[(2q)^3/3!]\sigma_{ooo}+\ldots \tag{20}$$

and

$$\sigma(\pm 2qV_e)=\pm 2q\sigma_e+[(2q)^2/2!]\sigma_{ee}\pm[(2q)^3/3!]\sigma_{eee}+\ldots. \tag{21}$$

These potentials are achiral in themselves, as is apparent from Eqs. (18) and (19). By taking superpositions of opposite charged potentials we may obtain σ_{oo}, σ_{ooo}, σ_{ee}, and σ_{eee}. Hence, all the expansion coefficients through third order may be obtained. Of course, to obtain the splittings and mean values we do not need all of the coefficients.

As we pointed out in Sec. I, we consider both Xe and a naked spin. The expansions are considered valid for both, as our calculations show.

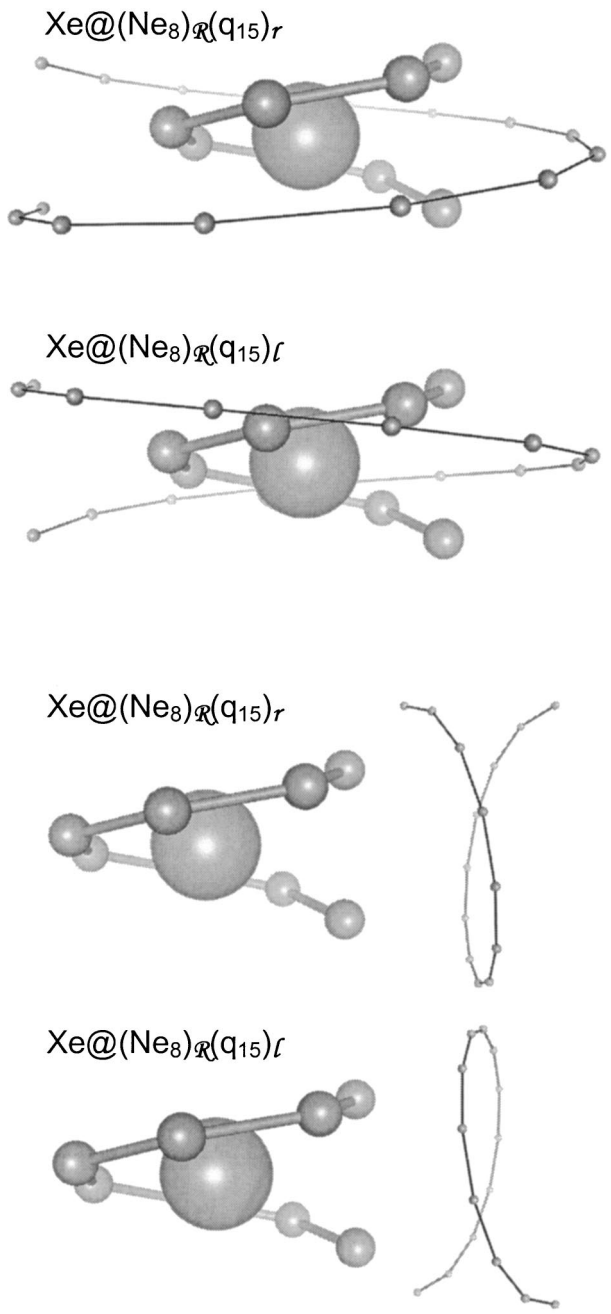

FIG. 1. Structures of model systems $Xe@Ne_8(\pm q)_{15}$ considered in this paper. A smaller radius was chosen for the charge helix in the orthogonal arrangement.

V. RESULTS AND DISCUSSION

The model system is a right-handed helix of 8 Ne atoms with an outer helix of 15 point charges. The top two pictures in Fig. 1 show the models used. *Ab initio* shielding calculations were carried out at four sets of values of charges $\pm q$ for Xe at the center of the Ne_8 helix (or a naked spin at the same location) employing methods described in detail in Ref. 1. The charge helix is coaxial with the Ne_8 helix. Magnitudes of charges used are 0, $0.123\,906e$, $0.061\,953e$, $0.030\,976e$, and $0.015\,488e$. The quantities that we need for the expansion in powers of q are the shielding values from which the shieldings for $q=0$ have been presubtracted; these are given in Table I for the naked spin and in Table II for Xe.

TABLE I. Calculated values of isotropic shielding (ppm) for the naked spin at the center of a Ne_8 helix and an outer helix of 15 identical partial charges, after presubtracting the $q=0$ value which is -0.0016 ppm.

Partial charge	$\sigma_{\mathcal{R}r}$ see Eq. (10)	$\sigma_{\mathcal{R}\ell}$ see Eq. (11)	Diastereomeric splitting $\sigma_{\mathcal{R}r}-\sigma_{\mathcal{R}\ell}$
+0.123 906	1.8447	1.8760	−0.0313
+0.061 953	0.9234	0.9425	−0.0191
+0.030 976	0.4620	0.4724	−0.0104
0.0	⋯	⋯	0
−0.030 976	−0.4627	−0.4748	+0.0121
−0.061 953	−0.9261	−0.9519	+0.0258
−0.123 906	−1.8553	−1.9137	+0.0584

From the values in Tables I or II we obtain the quantities $1/4[\sigma_{\mathcal{R}r}(+q)-\sigma_{\mathcal{R}\ell}(+q)-\sigma_{\mathcal{R}r}(-q)+\sigma_{\mathcal{R}\ell}(-q)]/q$, which when plotted against q^2 gives the intercept σ_o and the slope at the limit $q\to 0$ is $[\sigma_{ooo}+3\sigma_{eeo}]/3!$, according to Eq. (14). This plot is shown in Fig. 2(a), from which we obtain both parameters. On the other hand, the quantities $1/4[\sigma_{\mathcal{R}r}(+q)+\sigma_{\mathcal{R}\ell}(+q)-\sigma_{\mathcal{R}r}(-q)-\sigma_{\mathcal{R}\ell}(-q)]/q$ plotted against q^2 gives the intercept σ_e and the slope at the limit $q\to 0$ is $[\sigma_{eee}+3\sigma_{eoo}]/3!$, according to Eq. (15). This plot is shown in Fig. 2(b), from which we obtain both parameters. We also find σ_{eo} and $(\sigma_{ee}+\sigma_{oo})$ from applying Eqs. (16) and (17), respectively, that is, from the intercepts of the plots shown in Fig. 3. The values obtained for these parameters for Xe are shown in Table III. For the naked spin, we also obtain the parameters σ_e, σ_o, σ_{eo}, $(\sigma_{ee}+\sigma_{oo})$ from the same types of plots, neglecting the cubic terms. We need additional calculations to separate out the two derivatives that are included in the sums $(\sigma_{ee}+\sigma_{oo})$, etc.

The calculations of Xe shielding in a Ne_8 helix under the influence of superpositions of same and opposite charged potentials were carried out using superimposed left-handed and right-handed helices of charges, using sums and differences, that is

(i) $\sigma Xe@(Ne_8)_{\mathcal{R}}(+q_{15})_r(-q_{15})_\ell$,

(ii) $\sigma Xe@(Ne_8)_{\mathcal{R}}(-q_{15})_r(+q_{15})_\ell$,

TABLE II. Calculated values of isotropic shielding (ppm) for Xe at the center of a Ne_8 helix and an outer helix of 15 identical partial charges, after presubtracting the $q=0$ value which is 5573.9032 ppm.

Partial charge	$\sigma_{\mathcal{R}r}$ see Eq. (10)	$\sigma_{\mathcal{R}\ell}$ see Eq. (11)	Diastereomeric splitting $\sigma_{\mathcal{R}r}-\sigma_{\mathcal{R}\ell}$
+0.123 906	96.1699	99.5459	−3.3760
+0.061 953	48.0419	48.9761	−0.9342
+0.030 976	24.0093	24.3047	−0.2954
+0.015 488	12.0017	12.1085	−0.1068
0.0	⋯	⋯	0
−0.015 488	−11.9964	−12.0258	+0.0294
−0.030 976	−23.9858	−23.9744	−0.0114
−0.061 953	−47.9615	−47.6595	−0.3020
−0.123 906	−95.897	−94.3435	−1.5535

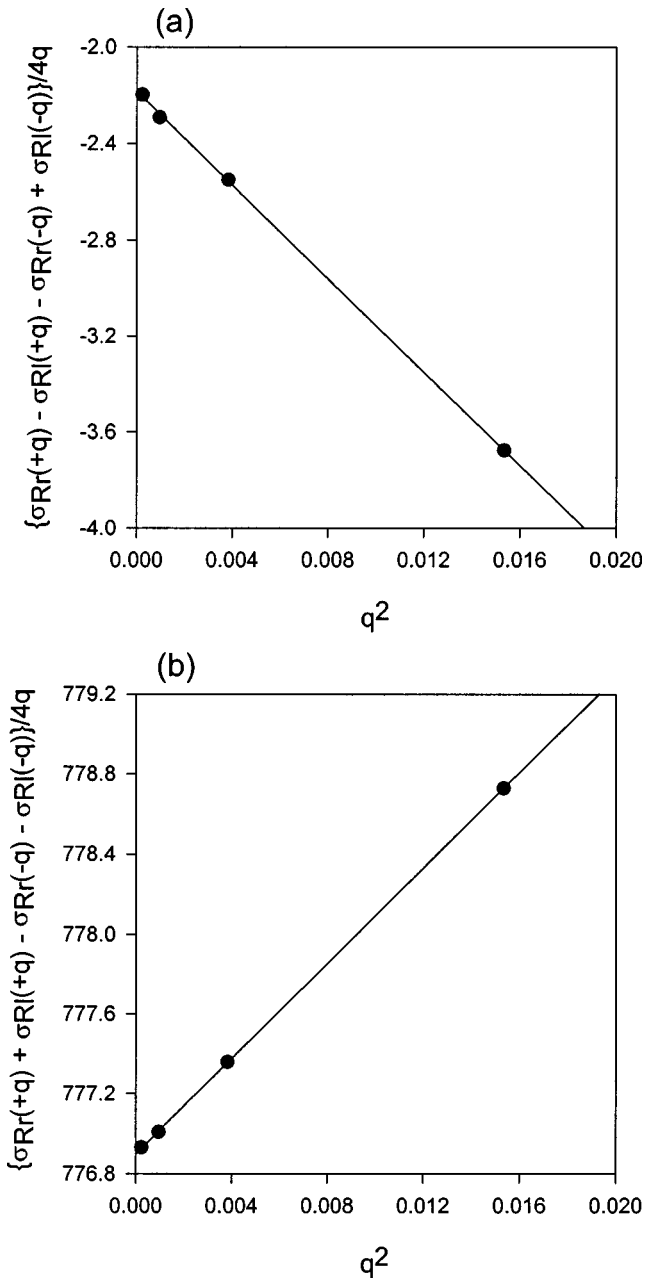

FIG. 2. The charge dependence of the Xe shielding provides values of the derivatives: (a) The intercept is σ_o, and the slope is $[\sigma_{ooo}+3\sigma_{eeo}]/3!$. (b) The intercept is σ_e, and the slope is $[\sigma_{eee}+3\sigma_{eoo}]/3!$. The values are given in Table III.

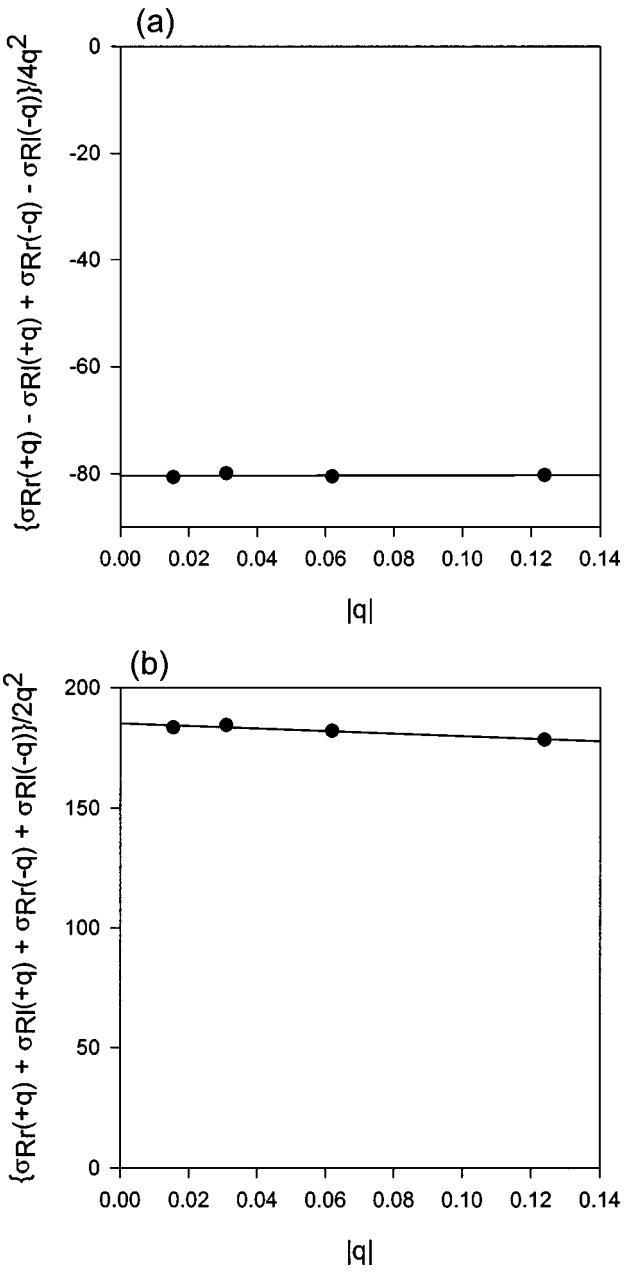

FIG. 3. The charge dependence of the Xe shielding provides values of the derivatives: In the lim $q\rightarrow 0$, the values are (a) σ_{eo} and (b) $\{\sigma_{ee}+\sigma_{oo}\}$. The values are given in Table III.

(iii) $\sigma\mathrm{Xe}@(\mathrm{Ne}_8)_{\mathcal{R}}(+q_{15})_r(+q_{15})_\ell$,

(iv) $\sigma\mathrm{Xe}@(\mathrm{Ne}_8)_{\mathcal{R}}(-q_{15})_r(-q_{15})_\ell$.

The results are shown in Table IV. Note that in Eq. (20) the $+$ sign corresponds to the combination $(+q_n)_r(-q_n)_\ell$ and the $-$ sign to $(-q_n)_r(+q_n)_\ell$.

From the values in Tables IV and II, we can separate out the combination of derivatives to obtain the individual derivatives up to order 3. The results are shown in Table V. Although the individual derivatives σ_{eoo} and σ_{eee} lack in precision, the combination $[\sigma_{eoo}+1/3\sigma_{eee}]=+238.4\pm 0.9\,\mathrm{ppm/e^3}$, is more accurately known. We have insufficient accuracy in the calculations involving superposition of oppositely charged helices (i) and (ii) to extract individual values for σ_{eeo} and σ_{ooo}, but the combination $[\sigma_{eeo}+1/3\sigma_{ooo}]=-194\pm 2\,\mathrm{ppm/e^3}$ is known.

The shielding at the naked spin is small and very nearly a linear function of charge. This is the molecular shielding of the Ne_8 helix changing under the influence of the chiral partial charge potential. Such a shielding would be present for any nucleus at this position. The Xe atom, however, has the additional shielding response which arises from the interaction between its electrons and the electrons of the Ne_8 helix. The Xe shielding response is larger and decidedly nonlinear with charge.

TABLE III. Parameters obtained from calculations at four sets of $\pm q$ values using Eqs. (10) and (11). See Figs. 2 and 3.

$\lim_{q=0}$	σ_o (ppm/e)	σ_e (ppm/e)	σ_{eo} (ppm/e^2)	$\{\sigma_{ee}+\sigma_{oo}\}$ (ppm/e^2)	$[\sigma_{eeo}+1/3\sigma_{ooo}]$ (ppm/e^3)	$[\sigma_{eoo}+1/3\sigma_{eee}]$ (ppm/e^3)
Naked	−0.1822	15.1070	+0.455	−1.680	⋯	⋯
spin	±0.0002	±0.0009	±0.0005	±0.002		
Xe	−2.184	776.90	−80.4	+185.1	−194	+238.4
	±0.008	±0.004	±0.3	±0.8	±2	±0.9

VI. SPATIAL CONFIGURATION OF THE DIASTEREOMERS AND THE DIASTEREOMERIC SPLITTING

The charge dependence of the diastereomeric splittings obtained from the *ab initio* calculations is shown in Tables I and II. In Fig. 4 we present the charge dependence of the diastereomeric splitting for the naked spin and the Xe system. Figure 4 shows that the individual derivatives used in Eq. (12) do accurately reproduce the diastereomeric splitting. That is, the expansion in q is sufficiently converged at third order for the magnitudes of charges used.

The diastereomeric splitting for the naked spin is small, it has nearly linear dependence on q, and it changes sign. That is, there is a switching of peak positions for $\mathcal{R}r$ and $\mathcal{R}\ell$ in the naked spin nuclear magnetic resonance (NMR) spectrum when the partial charges change systematically from large positive to large negative charge. On the other hand, we observe in Fig. 4 that the switching of the assignments of the two Xe peaks only occurs in the region of very small chiral potential. In all remaining regions, the Xe in $\mathcal{R}\ell$ is more shielded than in $\mathcal{R}r$, that is, same-handedness always appears at higher Xe chemical shift except where the perturbing potential is very weak. The even terms in the chiral potential provide a large chemical shift as much as 95 ppm in either direction for Xe, but the odd part of the shielding response, which is entirely responsible for the diastereomeric splitting, keeps the same-handed diastereomer at higher chemical shift than the opposite-handed diastereomer. That is, except where the partial charge helix provides too small a perturbation, in which case the molecular shielding of the Ne_8 helix (as seen by a naked spin) dominates and changes the sign of the diastereomeric splitting with the change in sign of the charge. Although the magnitude of the splitting depends on the magnitudes of the electrostatic potential, the chirality of the potential alone determines the sign of the diastereomeric splitting for Xe. In other words, for this configuration of diastereomers, unique assignment of the peaks to $\mathcal{R}\ell$ or $\mathcal{R}r$ is possible, except where the chiral potential is very weak, in which case the molecular shielding of the Ne_8 helix in the charge field can dominate the diastereomeric splitting and change sign with the sign of q.

We also carried out quantum mechanical calculations on another configuration of diastereomers. Instead of positioning the charge helix coaxially with the Ne_8 helix, we placed a charge helix with its axis orthogonal to the axis of the Ne_8 helix. The $\mathcal{R}r$ and $\mathcal{R}\ell$ diastereomers in this orthogonal configuration are shown as the two pictures at the bottom of Fig. 1. We found that for this perpendicular configuration, the diastereomeric splitting is smaller than for the coaxial configuration, and that the sign of the splitting is opposite that for the coaxial configuration. However, the significant result is that the sign of the diastereomeric splitting for the perpendicular configuration is, like the coaxial configuration, unchanged by replacing $+q$ with $-q$. In the perpendicular configuration, the Xe in $\mathcal{R}\ell$ is less shielded than in $\mathcal{R}r$, that is, same-handedness appears at lower Xe chemical shift, regardless of the sign of the charge. Thus, for a given spatial configuration of diastereomers, the assignment of the peaks to $\mathcal{R}\ell$ or $\mathcal{R}r$ appears to be reliable and independent of the sign or magnitude of the chiral potential.

The magnitude of the splitting and the absolute chemical shifts of Xe in the $\mathcal{R}r$ and in $\mathcal{R}\ell$ diastereomer do depend on the magnitude of the charges. However, we found that (except in very weak chiral potentials where the sign of the diastereomeric splitting switches with the sign of the partial charges as it does for the naked spin), the sign of the diastereomeric splitting for Xe is determined only by the chirality of the potential, for a given spatial configuration of diastereomers. Therefore, the unique assignment of the peaks in the Xe NMR spectrum to each of the $\mathcal{R}r$ and $\mathcal{R}\ell$ diastereomers is theoretically possible, even when the second chiral body, or center, is represented entirely by a chiral potential rather than groups of atoms, provided the calculations are carried out in the correct spatial configurations.

TABLE IV. Results of *ab initio* calculations of Xe shielding in superposition of left- and right-handed charge helices, after subtracting the Xe shielding in $Xe@(Ne_8)_{\mathcal{R}}$ which is 5573.9032 ppm.

$\lvert q\rvert$	(i) $Xe@(Ne_8)_{\mathcal{R}}$ $(+q_{15})_r(-q_{15})_\ell$	(ii) $Xe@(Ne_8)_{\mathcal{R}}$ $(-q_{15})_r(+q_{15})_\ell$	(iii) $Xe@(Ne_8)_{\mathcal{R}}$ $(+q_{15})_r(+q_{15})_\ell$	(iv) $Xe@(Ne_8)_{\mathcal{R}}$ $(-q_{15})_r(-q_{15})_\ell$
0.015 488e	−0.0678	+0.0671	+19.3768	−19.2583
0.030 976e	−0.1364	+0.1334	+38.8775	−38.4051
see Eq.	(20)+	(20)−	(21)+	(21)−

TABLE V. Derivatives of Xe shielding with respect to charge (ppm/e^n).

σ_o	σ_e	σ_{eo}	σ_{oo}	σ_{ee}	$\sigma_{eeo}+1/3\sigma_{ooo}$	$\sigma_{eoo}+1/3\sigma_{eee}$	σ_{eoo}	σ_{eee}
-2.184	$+776.9$	-80.4	-62.5	$+246.6$	-194	$+238.4$	$+(8\pm5)$	$-(24\pm15)$
±0.008	±0.004	±0.3	±1	±0.6	±2	±0.9	$\times10^5$	$\times10^5$

VII. APPLICATION TO Xe IN TETHERED CAGES

Now we address the application of our model system to tethered cages. We have shown that the diastereomeric splitting depends only on the odd terms in the chiral potential (the odd powers of V_{odd}). This is general, whether the potential arises from the handed tethers with all their electrons and nuclei or from only partial charges representing the tether atoms. If our model system has served us well, we may assume that the chiral potential from bare charges alone is sufficient to model the diastereomeric aspects of the handed tethers attached to the chiral cryptophane cage. In the same way that the Xe shielding response from the Ne_8 helix includes the interaction between the Ne_8 helix and the chiral potential of the charge helix, we assume that the Xe shielding response from the cryptophane cage atoms will include the chiral potential of the tether, when the tether is only represented by bare charges.

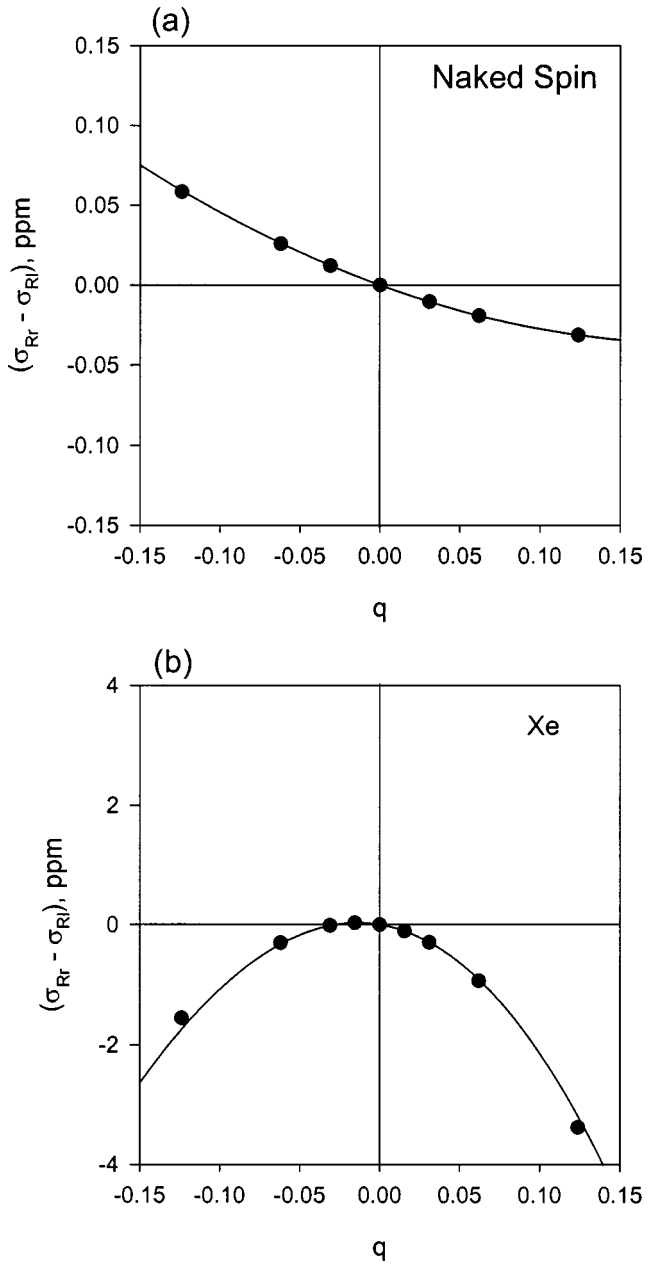

FIG. 4. The dependence of the diastereomeric splitting on the charge, for (a) the naked spin, and (b) for Xe. The curves drawn demonstrate that the diastereomeric splittings can be reproduced by Eq. (12) using the derivatives in ppm/e^n: $\sigma_o=-0.1822\pm0.0002$, $\sigma_{eo}=+0.455\pm0.0005$ for the naked spin, and $\sigma_o=-2.184\pm0.008$, $\sigma_{eo}=-80.4\pm0.3$, $\sigma_{eeo}+1/3\sigma_{ooo}=-194\pm2$, for Xe.

In considering two different spatial configurations of diastereomers (coaxial and perpendicular helices), we found that, while the magnitude of the diastereomeric splitting depends on the magnitude and sign of the charges, the chirality of the potential alone determines the relative order of the chemical shifts of Xe in $\mathcal{R}r$ and in $\mathcal{R}\ell$ diastereomers for a given configuration of diastereomers. On the basis of what we have found, *it is possible for calculations in the chiral potential of the tethers (represented by partial charges in lieu of atoms) to predict the assignments of the* Xe NMR *signals to the respective* $\mathcal{R}r$ *and* $\mathcal{R}\ell$ *diastereomers of the tethered cryptophane cages, provided we carry out the calculations in the correct spatial configurations.*

VIII. CONCLUSIONS

In our first two papers[1,2] we investigated the relation between nuclear magnetic shielding and the chirality of the systems being measured. The chiral systems were represented by a Xe atom or a naked spin embedded in a helix of Ne atoms. Because shielding experiments are unable to directly measure chirality, we created diastereomers by further embedding the Xe/naked spin complexes in coaxial helices of point partial charges of a given sign and strength. In this paper, the final one in our series, we examine the explicit role played by the structure of the potential evinced by the partial charge helix.

We have established that the shielding can be analyzed in terms of a potential whose chirality is explicitly exhibited. The shielding may be written as a perturbation expansion in charge for both positive and negative charges. By realizing that the difference between ℓ and r helices is only in the sign of V_o, we are able to determine the terms in the expansion, all of them to a given order. The individual derivatives of the shielding with respect to the charge may be obtained by considering the terms in the expansion for the superpositions of the potentials.

As expected, the difference of the chemical shift be-

tween an $\mathcal{R}\ell$ and an $\mathcal{R}r$ complex—the diastereomeric splitting—only depends upon odd powers of the odd portion of the chiral potential. The difference between the splittings of Xe diastereomers and equivalent naked spin diastereomers is profound. The splitting of the chemical shifts of the naked spin diastereomers is linear to quite large values of the partial charge. Hence, a linear response theory in V_o is valid. That is, the naked spin chiral splitting does not know the difference between a given odd potential and that which arises from the odd portion of a chiral potential. On the other hand, the splitting of the Xe diastereomers is strongly quadratic in the partial charge, as well as linear. Except in very weak chiral potentials where the sign of the splitting switches with the sign of the partial charges as it does for the naked spin, the sign of the Xe diastereomeric splitting in moderate chiral potentials is the same for both positive and negative partial charges. This means that the chiral nature of the partial charge helix is fully felt by the Xe nuclear spin. The nonlinear polarization of the Xe electrons coupled to the Ne helix swamps the linear response. When embedded in partial charge helices in a different geometry, again, the sign of the splitting was the same for both signs of the partial charge. But the sign was opposite that of the coaxial partial charge system with the same magnitude of partial charge strength. Hence, for Xe both geometry and chirality matter.

The grand conclusion is that Xe imbedded in a given chiral molecule is remarkably sensitive to the presence of further chiral systems. The splitting of diastereomeric nuclear shielding elements are acute measures of chirality. In the real molecular system of interest, the tethers therefore act as probes of the chirality of Xe@cage.

With these findings, we expect that diastereomeric splittings arising from real tethers can be modeled by *ab initio* calculations of Xe@cage in the presence of differently oriented partial charges. Adding different functional groups to the tether will give splittings commensurate with independent tethers as given potentials in Hartree–Fock or Kohn–Sham equations. The assumption is that a self-consistency between tethers and Xe@cage might not be necessary for Xe shielding calculations to provide assignments of individual Xe signals to diastereomers.

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