# Valence Bond Studies of Internuclear Coupling

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Two studies are reported which involve valence bond calculations of internuclear coupling. The first, of the proton spectra for the —CH<sub>2</sub>CH<sub>2</sub>— bridges in (2, 2) metacyclophane, shows conclusively that the relative signs of the geminal and vicinal proton coupling constants are opposite, which disagrees with the theoretical prediction that both are positive. In this compound, the C—CH<sub>2</sub>—CH<sub>2</sub>—C groups are locked in position with the dihedral angle between alkyl C—C—C bonds slightly less than the symmetrical, staggered 60°. A complete analysis of the A<sub>2</sub>X<sub>2</sub> and A<sub>2</sub>B<sub>2</sub> type proton spectra, at 60 and 15 Mc/sec leads to the following assignments:  $J_t^{HH} = \pm 12.3$ ,  $J_g^{HH}$  (the coupling of the "central" pair of gauche protons) =  $\pm 3.2$ ,  $J_{gem}^{HH} = \mp 12.0$ , and  $J_g^{HH} = \pm 4.0$ , all  $\pm 0.1$  c/sec. A second related study is concerned with an interpretation for the additivity of substituent contributions to the <sup>13</sup>C—H coupling constant. Each atom or group X is assigned a characteristic "affinity" for s character in the carbon hybrid orbital of the C—X bond. The additivity can be derived if the s character is distributed among the four carbon orbitals in accord with the relative s affinities of the four substituents, provided that the total s character is conserved. The valence bond approach used with this model gives a linear relation between the s character of the carbon hybrid orbital involved in a C—H bond and the observed <sup>13</sup>C—H coupling constant ( $J_{CH} = 500 \alpha_{H}^{2}$ ).

Valence-bond methods have been used to calculate internuclear coupling constants for non-bonded 1 and also for directly bonded nuclei.2 For non-bonded nuclei, the σ-electron contribution to the coupling has been expressed in terms of deviations of the molecular electronic structure from perfect pairing.<sup>1</sup> Such calculations for protons predicted the geminal coupling  $J_{\text{gem}}^{\text{HH}}$  to be +12.5 c/sec in methane,<sup>1</sup> and subsequent, more approximate, calculations <sup>3</sup> for vicinal protons in the HCCH ethanic fragment gave the trans coupling  $J_t^{HH}$  to be about +9.2 c/sec and the gauche  $J_{\theta}^{HH}$ , +1.7 c/sec. These magnitudes agree well with experiment except that the trans vicinal constants observed for ethanic groups 4 (and also the cis and trans constants for ethylene 3) are often about 50% larger than predicted. Usually, only the magnitudes of J have been obtained from experiment; but increasing attention is being given to the importance of their relative signs. Several substituted ethylenes have been reported 5, 6 in which the sign of  $J_{\text{gem}}^{\text{HH}}$  (1 to 3 c/sec) is opposite to, and also the same as, that of  $J_{\text{cis}}^{\text{HH}}$  (5 to 11 c/sec) and  $J_{\text{trans}}^{\text{HH}}$  (12 to 18 c/sec). These results are compatible with the valence-bond calculations for the CH2 fragment,5 which neglect substituent and  $\pi$ -electron effects, and which predict that  $J_{\rm gem}^{\rm HH}$  should become negative for HCH bond angles larger than about  $120^{\circ}.5$ , 7 Similarly, there are substituted ethanes in which the sign of the vicinal coupling  $J_g^{\rm HH}$  (1 to 3 c/sec) is opposite to and also the same as, that of  $J_t^{\rm HH}$  (10 to 16.5 c/sec).4, 6 Again, the results seem compatible with the calculated dependence of the coupling upon the dihedral angle  $\phi$ ,

$$J_{\text{vic}}^{\text{HH}} \simeq 9 \cos^2 \phi - 0.3,\tag{1}$$

because substituent effects were neglected and tetrahedral HCC angles were assumed.<sup>3</sup> A more troublesome question has been raised by relative sign determinations, in diethyl sulphite <sup>8</sup> and in several dioxolane derivatives,<sup>9</sup> which conflict with the predictions <sup>1, 3</sup> that large values of  $J_{\rm gem}^{\rm HH}$  and  $J_{\rm vic}^{\rm HH}$  should both be positive. However, the

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compounds studied are such that substituent effects, angular distortions and motional averaging are important, and their neglect in the theoretical treatment might be responsible for the apparent discrepancies in the relative signs. Therefore, we have made a detailed study of the proton spectrum of the —CH<sub>2</sub>CH<sub>2</sub>— groups in (2, 2) metacyclophane, <sup>10</sup> the conformation of which is given in fig. 1. This compound avoids the uncertainties of the cases reported earlier <sup>8, 9</sup> because an x-ray structural determination of the solid <sup>11</sup> has shown that the methylene groups are locked in virtually symmetrical, staggered positions, with tetrahedral bond angles. Nonetheless, opposite signs are found for the large trans and geminal constants, in agreement with the previous experiments <sup>8, 9</sup> and disagreeing with the theoretical predictions that both are positive.<sup>1, 3</sup> This disagreement may result either from inaccurate molecular wave functions or from the approximations made in calculating the coupling of the non-bonded nuclei, and both aspects require further theoretical study.

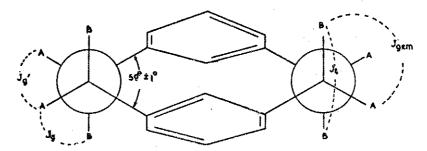


Fig. 1.—The structure of (2, 2) metacyclophane and the conformation of the —CH<sub>2</sub>CH<sub>2</sub>— "bridges" whose proton spectra were analyzed. The protons in the —CH<sub>2</sub>CH<sub>2</sub>— groups are labelled A and B, and symbols are defined for the coupling constants.

Thus far, the valence bond calculations for directly bonded nuclei appear to be more reliable. In this case, deviations from perfect pairing are relatively unimportant, and further simplification results when the coupling depends mainly on the Fermi contact term as in the  $^{13}$ C—H group.<sup>2</sup> A number of theoretical and experimental studies indicate that  $J_{\rm CH}$  is determined by the carbon orbital hybridization and by the polarity of the C—H bond.<sup>2, 12, 13</sup> In fact  $J_{\rm CH}$  has been employed as a simple measure of orbital hybridization. More recently, attention has been turned to the effects of substituents upon  $J_{\rm CH}$ , and several interesting empirical relationships have been discovered, <sup>14, 15</sup> the most basic of which is probably the linear additivity of group contributions to  $J_{\rm CH}$  in substituted methanes.<sup>14</sup> We have found that this relation can be derived by assuming that a substituent changes the hybridization of the carbon 2s orbital in a characteristic fashion.<sup>16, 17</sup>

Substituent effects have also been noted for H—H coupling in hydrocarbons. In particular, more or less linear relations have been found between the electronegativity of the substituent and the geminal and/or vicinal coupling constants in substituted ethylenes  $^{18-20}$  and ethanes. As yet, no detailed, theoretical interpretation of these effects appears to have been advanced. However, it seems very probable that the effects of substituents upon  $J_{\rm CH}$  are related directly, or at least indirectly, to those for  $J_{\rm HH}$ . If our model is correct for the effect of X upon  $J_{\rm CH}$  in CHXYZ or CH<sub>2</sub>—CHX groups, it should contribute to a better understanding of  $J_{\rm HH}$ , inasmuch as the latter is also affected by the hybridization of orbitals in the C—H bonds.

# RELATIVE SIGNS OF $J_{\mathrm{gem}}^{\mathrm{HH}},~J_{\mathrm{g}}^{\mathrm{HH}}$ and $J_{t}^{\mathrm{HH}}$

A sample of (2, 2) metacyclophane was provided for our experiments by Wilson, Boekelheide, and Griffin.<sup>10</sup> The high resolution proton spectra were observed at

room temperature using 10% solutions in CCl<sub>4</sub>. Spectra at 60 Mc/sec were observed with Varian Associates HR-60 and A-60 spectrometers. The 15·083 Mc/sec spectrum was obtained through the courtesy of Dr. J. N. Shoolery at Varian Associates, where it was observed with a V-4300 spectrometer system.

The general procedure used to determine the magnitudes and relative signs of the coupling constants in the — $CH_2CH_2$ — group is the following.<sup>22</sup> At a resonance frequency of 60 Mc/sec, the chemical shift  $v_0\delta$  between the  $A_2$  and  $B_2$  sets of protons, defined in fig. 1, is sufficiently large that the quite simple observed spectrum is a good approximation to the  $A_2X_2$  type. From it, the magnitudes of  $v_0\delta$  and of the four coupling constants are determined readily, as well as the relative signs for each of two pairs of coupling constants. In part, the 60 Mc/sec spectrum is easy to analyze because it is insensitive to one of the relative signs. However, the latter becomes important at lower resonance frequencies, where the spectrum is of the  $A_2B_2$  type. Therefore, the magnitudes and signs obtained from the 60 Mc/sec spectrum were used to calculate 15.083 Mc/sec spectrum for the remaining relative sign permutations, and comparison of these with the observed spectrum completes the analysis.

It is convenient to use the parameters

$$K = \pm |J_t + J_{g'}|, \qquad N = |J_g + J_{\text{gem}}|,$$

$$M = |J_t - J_{g'}|, \qquad L = |J_g - J_{\text{gem}}|,$$

$$(2)$$

where the coupling constants are defined in fig. 1. These four constants have three relative signs which we wish to establish. In terms of the parameters K, L, M and N, which we treat as positive quantities except for K in the one circumstance noted below, the relative signs of each pair of coupling constants in eqn. (2) is determined by the relative values of the corresponding two parameters. Thus, if N>L,  $J_g$  and  $J_{gem}$  have the same sign; and if N < L, the opposite. Identical relations involving K and Mhold for  $J_t$  and  $J_{g'}$ . In addition, the spectrum is sensitive to the actual relative signs of  $K = (J_t + J_{g'})$  and  $N = (J_g + J_{gem})$ . Whether the observed spectrum is fitted by K positive or negative, while treating N as positive, determines the third relative sign. If K negative applies, then the constant of largest magnitude in K is of opposite sign to the constant of largest magnitude in N, while they are of the same sign for a positive K. Finally, the magnitudes of the coupling constants are obtained by means of eqn. (2) from the numerical (positive) values for K, L, M, and N; however, the spectrum alone does not tell which constant is which within each pair and supplemental information about the relative magnitudes of the constants is required to complete the assignment.

## THE 60 Mc/sec SPECTRUM

The proton spectrum observed at 60 Mc/sec is given in fig. 2. As a first approximation it is of the  $A_2X_2$  type, with "mirror image"  $A_2$  and  $X_2$  multiplets whose centres are separated by  $[(\nu_0\delta)^2 + N^2]^{\frac{1}{2}}$ . In general, each  $A_2X_2$  multiplet has ten lines, two quartets and a doublet with a common centre. The outer splitting of one quartet is K, and of the other, M, while the central splittings are  $(K^2 + L^2)^{\frac{1}{2}} - K$  and  $(M^2 + L^2)^{\frac{1}{2}} - M$ , respectively. The lines of the doublet are the strongest transitions; their splitting is N. In the observed spectrum, the  $A_2$  and  $A_2$  multiplets have two rather broad, very strong lines at the centre, with two weaker lines at each side. Therefore, the inner lines of the two quartets are not resolved from the strong N-doublet, and only the outer lines of the quartets are visible. Thus, the  $\sim 8$  c/sec splitting of the strong centre pair of lines undoubtedly is N. Also, the outer splittings of the two quartets are  $\sim 9$  and  $\sim 15$  c/sec but at this point it is uncertain which is K and which is M. These values, in combination with the expressions for the central

splitting of the two quartets and their observed values of  $\sim 8$  c/sec, give an unambiguous value for L of  $15\pm 2$  c/sec. Also, the separation between the centres of the two multiplets is approximately the chemical shift, which gives  $v_0\delta = 60.3$  c/sec.

The values of the parameters were refined by varying them systematically, comparing the resulting calculated spectra  $^{23}$  with experiment, and then interpolating. In this manner, the following best-fit, numerical values were obtained:  $v_0\delta = 59\cdot 1$ ,  $N = 8\cdot 0$ ,  $L = 16\cdot 0$ , and  $\pm K$  or  $M = 9\cdot 1$  or  $15\cdot 5$ , all in c/sec. More important, the spectra calculated for the four possible permutations show that although the spectrum observed at 60 Mc/sec is too insensitive to the sign of K for its determination, the asymmetry in the splittings p and q in fig. 2 is governed by the relative magnitudes of K and M. In order to have p < q as observed, it is necessary to have K > M,  $^{22}$  which requires that K be 15·5 and M, 9·1 c/sec.

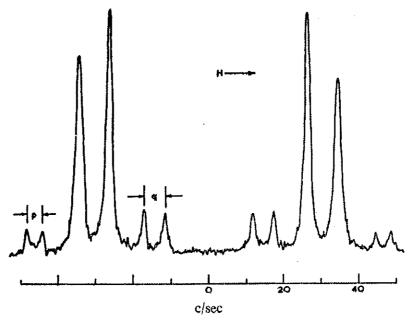


Fig. 2.—The spectrum observed at 60 Mc/sec for the —CH<sub>2</sub>CH<sub>2</sub>— group protons in (2, 2) metacyclophane. This spectrum is fitted by  $v_0\delta = 59\cdot 1$ ,  $N = 8\cdot 0$ ,  $L = 16\cdot 0$ ,  $M = 9\cdot 1$  and  $\pm K = 15\cdot 5$  c/sec. Spectra calculated for interchanged values of K and M have p > q, rather than p < q as observed.

### THE 15.083 Mc/sec SPECTRUM

Figure 3 includes the spectrum observed at 15.083 Mc/sec and also spectra calculated for the two remaining sign permutations,  $K = \pm 15.5$  c/sec. There is excellent agreement between experiment and the spectrum calculated for K = -15.5 c/sec, and very poor agreement for K = 15.5 c/sec. Therefore, the parameters which apply to the —CH<sub>2</sub>CH<sub>2</sub>— group are:

$$K = -15.5 \text{ c/sec}, \qquad N = 8.0 \text{ c/sec},$$
 $M = 9.1, \qquad L = 16.0.$  (3)

Upon combining these results with the definitions in eqn. (2) we find from N and L that  $J_g$  and  $J_{gem}$  are  $12\cdot0$  and  $4\cdot0$  or  $4\cdot0$  and  $12\cdot0$  c/sec. Moreover, they are of opposite signs because N < L. From K and M,  $J_t$  and  $J_{g'}$  are  $12\cdot3$  and  $3\cdot2$  c/sec or the reverse. Also, they are of the same sign because K > M. (Here, both K and M must be treated as positive quantities.) Also K and N actually have opposite signs so the largest constant of the K, M pair ( $12\cdot3$  c/sec) is of opposite sign to the largest constant of the N, L pair ( $12\cdot0$  c/sec).

The assignment is completed by introducing the inequality  $|J_t| > |J_{g'}|$ , which is known with certainty from the nmr studies of substituted ethanes,<sup>4, 6</sup> and the inequality  $|J_{\text{gem}}| > |J_g|$  which is equally certain from the experimental results <sup>4, 6</sup> for substituted ethanes in combination with those on  $J_{\text{gem}}$  in methane <sup>1</sup> and substituted methanes.<sup>5</sup> The final assignment is

$$J_t^{\rm HH} = \pm 12.3 \text{ c/sec}, \qquad J_{\rm gem}^{\rm HH} = \mp 12.0 \text{ c/sec}, 
J_{g'}^{\rm HH} = \pm 3.2, \qquad J_g^{\rm HH} = \pm 4.0$$
(4)

with probable errors of about  $\pm 0.1$  c/sec in the numerical values.

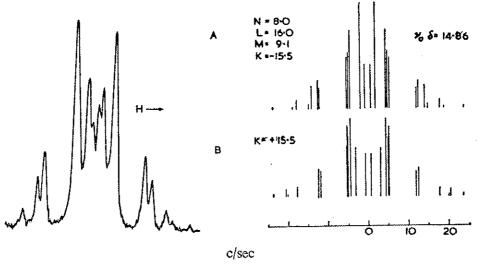


Fig. 3.—The left-hand spectrum is that observed at 15.083 Mc/sec for the —CH<sub>2</sub>CH<sub>2</sub>— group protons in (2, 2) metacyclophane. The line spectra at the right were calculated for the two sign permutations not differentiated by the 60 Mc/sec spectrum, i.e. for  $K = \pm 15.5$  c/sec. The spectrum for K positive disagrees with the wings and the central portion of the observed spectrum.

#### COMMENTS

The closeness of the  $12.0\pm0.1$  c/sec value found for  $J_{\text{gem}}^{\text{HH}}$  to the  $12.4\pm0.6$  c/sec observed in methane 1 indicates that the former is not affected by angular distortion and substituent effects. The small difference between the 3.2 and 4.0 c/sec values for  $J_{q'}^{HH}$  and  $J_{q}^{HH}$  is consistent with a C—C—C—C dihedral angle of slightly less than the 60° for a symmetric, staggered —CH2CH2— group, as is suggested by the X-ray data for the solid.11 Also, this could account for the value of 12.3 c/sec for Jih being smaller than most found for substituted ethanes.6,7 Therefore, our finding of large values of opposite sign for  $J_t^{HH}$  and  $J_{gem}^{HH}$ , as well as the less conclusive earlier studies, 8, 9 show that either the calculation on CH<sub>4</sub> 1 or that on the ethanic (and probably also on the ethylenic) fragment 3 is in error. Which of the calculations is most likely to be in error, if not both, is another question. In some ways, the calculations for the HCCH fragment present the best opportunities for error. These calculations are more complex than for CH<sub>4</sub> (or CH<sub>2</sub>), and it is possible for example that the non-neighbouring-atom exchange integrals should not have been neglected.3 A more direct approach to the question would be to determine the sign of  $J_{\text{vic}}^{\text{HH}}$  and/or  $J_{\rm gem}^{\rm HH}$  with respect to  $J_{\rm CH}$  for there is little doubt but that it is positive.<sup>2, 17</sup> Such relative sign determinations could help decide which of the  $J_{\rm HH}$  calculations to redo first. Fortunately, the relative signs of  $J_{\rm CH}$ ,  $J_{\rm gem}^{\rm HH}$  and  $J_{\rm vic}^{\rm HH}$  can be determined by the sort of approach used here and also by double resonance methods, either on 13C enriched (2, 2) metacyclophane or other appropriate compounds.

In fact, analysis of the  $^{31}P$  and proton spectra observed for diphosphine  $H_2PPH_2$  has given results,  $^{24}$  related to our problem. For this compound,  $J_{\rm gem}^{\rm HH}$  and  $J_{\rm PP}$  were found to have values of  $108\cdot2$  and 12 c/sec, respectively, and to be of the same sign, opposite to that of  $J_{\rm vic}^{\rm HH}$  (cis and trans) which has values of  $10\cdot5$  and  $6\cdot8$  c/sec. By analogy to the results of the HCCH calculations,  $^3$  it was assumed  $^{24}$  that  $J_{\rm vic}^{\rm HH}$  was positive in diphosphine, which, of course, made  $J_{\rm PP}$  and  $J_{\rm gem}^{\rm HH}$  negative. A negative value for  $J_{\rm PP}$  is surprising because the coupling between directly bonded atoms due to the usually dominant contact term is positive. In view of the present findings it may be somewhat more plausible to take  $J_{\rm PP}$  as positive, which leads to  $J_{\rm gem}^{\rm HH}$  positive and  $J_{\rm vic}^{\rm HH}$  (cis and trans) negative, at least in the diphosphine case.

## EFFECTS OF SUBSTITUENTS UPON $J_{\mathrm{CH}}$

Malinowski has reported <sup>14</sup> that to a very good approximation the <sup>13</sup>C—H coupling constant in substituted methanes, CHXYZ, is an additive property of the substituents. This additivity has been expressed in two equivalent forms <sup>14, 17</sup> employing different definitions of the "substituent parameters". What is perhaps a better formulation may be obtained by returning to the basic experimental fact, namely, <sup>14, 17</sup>

$$J_{\text{CH}}(\text{CHXYZ}) = J_{\text{CH}}(\text{CH}_3\text{X}) + J_{\text{CH}}(\text{CH}_3\text{Y}) + J_{\text{CH}}(\text{CH}_3\text{Z}) - 2J_{\text{CH}}(\text{CH}_4),$$
 (5)

and noting that it may be written as

$$J_{\text{CH}}(\text{CHXYZ}) = J_{\text{CH}}(\text{CH}_4) + \delta_{\text{X}} + \delta_{\text{Y}} + \delta_{\text{Z}}, \tag{6}$$

where, by definition

$$\delta_{\mathbf{X}} = J_{\mathbf{CH}}(\mathbf{CH}_{3}\mathbf{X}) - J_{\mathbf{CH}}(\mathbf{CH}_{4}). \tag{7}$$

In other words, each substituent X contributes a characteristic term  $\delta_X$ , to  $J_{CH}(CHXYZ)$ , which is independent of the other substituents.

There are two general approaches to the theoretical interpretation of this empirical result. Previous work  $^{2, 12, 13}$  is consistent with  $J_{\rm CH}$  being determined by the carbon orbital hybridization and the C—H bond polarity. Therefore, one can seek to derive eqn. (6) on the basis of hybridization and/or polarity changes produced in the C—H bond by the substituent. Or one can investigate the other contributions, such as  $\pi$ -electron and orbital polarization terms, which X could make to  $J_{\rm CH}$  without affecting materially the C—H bond. We are concerned here with the first approach.

# VALENCE BOND FORMULATION FOR $J_{ m CH}$

The general expression for  $J_{\rm NN}$  consists of several terms.<sup>25</sup> However, in this paper we consider only the Fermi contact term which is dominant for the <sup>13</sup>C—H coupling, at least in CH<sub>4</sub>,<sup>2</sup>

$$J_{\text{CH}} \approx (J_{\text{CH}})_{\text{contact}} = \frac{-2}{3\hbar\Delta E} \left(\frac{16\pi\beta\hbar}{3}\right)^2 \gamma_{\text{C}} \gamma_{\text{H}} \left(\Psi_0 \mid \sum_{k,j} \delta(\mathbf{r}_{k\text{C}}) \delta(\mathbf{r}_{j\text{H}}) \mathbf{S}_k \cdot \mathbf{S}_j \mid \Psi_0\right). \tag{8}$$

The symbols used above have their usual meanings. In the ground state wave function  $\Psi_0$  deviations from perfect pairing are not important for the coupling of directly bonded nuclei.<sup>2</sup> We use the separated electron pair wave function,

$$\Psi_0 = (8!)^{-\frac{1}{2}} \sum_{P} (-1)^P P \left[ \Psi_{ax}(1,2) \psi_{by}(3,4) \psi_{cz}(5,6) \psi_{dh}(7,8) \right], \tag{9}$$

with

$$\psi_r(i,j) = u_r(i,j) \left( \frac{\alpha(i)\beta(j) - \beta(i)\alpha(j)}{\sqrt{2}} \right), \tag{10}$$

where  $u_r(i, j)$  is of the valence bond form with inclusion of ionic terms,

$$u_1(1,2) = \eta_1(\phi_a(1)\phi_x(2) + \phi_a(2)\phi_x(1) + \lambda_a\phi_a(1)\phi_a(2) + \lambda_x\phi_x(1)\phi_x(2)). \tag{11}$$

In the latter,  $\phi_a$ , ...  $\phi_d$  are carbon atomic orbitals;  $\phi_x$ , ...  $\phi_h$  are atomic orbitals on the atoms bonded to the carbon, and  $\eta$  is the normalization constant. The coefficients of the ionic terms are  $\lambda_a$  and  $\lambda_x$ . Substituting  $\Psi_0$  into eqn. (8) and using the Dirac identity  $S_k \cdot S_j = (\frac{1}{4})(2P_{kj}^s - 1)$ , in which  $P_{kj}^s$  is an operator interchanging the spins of electrons k and j, one obtains

$$J_{\text{CH}} = \frac{\gamma_{\text{C}}\gamma_{\text{H}}}{\hbar\Delta E} \left(\frac{16\pi\beta\hbar}{3}\right)^{2} \eta^{2} (\phi_{d} \mid \delta(\mathbf{r}_{i\text{C}}) \mid \phi_{d}) (\phi_{h} \mid \delta(\mathbf{r}_{j\text{H}}) \mid \phi_{h}). \tag{12}$$

We assume the four carbon hybrid orbitals to be formed from one 2s orbital and three 2p orbitals, e.g.

$$\phi_d = \alpha_H s + (1 - \alpha_H^2)^{\frac{1}{2}} p_\sigma \text{ and } \phi_a = \alpha_X s + (1 - \alpha_X^2)^{\frac{1}{2}} p_{\sigma'},$$
 (13)

where the s character,  $\alpha_{\rm H}^2$ ,  $\alpha_{\rm X}^2$ , etc., of the orbitals depends on the groups or atoms H, X, Y or Z bonded to the carbon. Substituting  $\phi_d$ , and  $\phi_h \equiv 1s_{\rm H}$  into eqn. (12), one finds that

$$J_{\text{CH}} = \frac{\gamma_{\text{C}} \gamma_{\text{H}}}{\hbar \Delta E} \left( \frac{16\pi \beta \hbar}{3} \right)^2 \eta^2 \alpha_{\text{H}}^2 \left| 2s_{\text{C}}(0) \right|^2 \left| 1s_{\text{H}}(0) \right|^2, \tag{14}$$

where

$$\eta^{-2} = (2 + (2 + \lambda_{\rm C} \lambda_{\rm H}) [\alpha_{\rm H}^2 S_s^2 + (1 - \alpha_{\rm H}^2) S_p^2 + 2\alpha_{\rm H} (1 - \alpha_{\rm H}^2)^{\frac{1}{2}} S_s S_p] + 4(\lambda_{\rm C} + \lambda_{\rm H}) \times [\alpha_{\rm H} S_s + (1 - \alpha_{\rm H}^2)^{\frac{1}{2}} S_p] + \lambda_{\rm C}^2 + \lambda_{\rm H}^2).$$
(15)

 $2s_{\rm C}(0)$  is the 2s wave function of carbon evaluated at the carbon nucleus, and  $1s_{\rm H}(0)$  is the corresponding quantity for the hydrogen 1s function.  $S_s$  and  $S_p$  are the overlap integrals between the hydrogen 1s atomic orbital and the 2s and 2p carbon atomic orbitals, respectively. In eqn. (15) for  $\eta^{-2}$ ,  $\lambda_{\rm H}$  is much less than  $\lambda_{\rm C}$ , because the electronegativity of C is greater than that of H, so  $\lambda_{\rm H}$  is neglected and the coefficient of the ionic contribution to the wave function is hereafter denoted by  $\lambda_{\rm C-H}$ .

Eqn. (14) leads to

$$J_{\rm CH} = (A\eta^2/\Delta E)\alpha_{\rm H}^2 \equiv J_0\alpha_{\rm H}^2 \text{ c/sec}, \tag{16}$$

where A is a collection of constants, and  $J_0$  is 500 c/sec, as determined from the observed value <sup>12, 13</sup> of 125 c/sec for  $J_{\rm CH}({\rm CH_4})$ . This value for  $J_0$  is consistent with the valence bond theory inasmuch as Karplus and Grant 2 obtained a reasonable value of 0.374 for  $\lambda_{\rm C-H}$ , using the same approach, with  $J_{\rm CH}=124$  c/sec, in combination with an estimate of  $\Delta E$  and calculations of the overlap integrals from Hartree-Fock functions. Eqn. (16), depending upon the sensitivity of  $\Delta E$  and  $\eta^2$  to substituents, affords an attractive semi-empirical way to obtain the s character of bonding orbitals from coupling constants. For the substituted methanes, or other classes of closely related compounds, one would expect  $\Delta E$  to be very nearly constant. This follows from the fact that it is approximately twice the bond energy,<sup>2</sup> which varies by only a few percent for C—H bonds. The constancy of  $\eta^2$  depends upon its sensitivity to  $\lambda$  and  $\alpha_H^2$ . These dependences can be calculated relatively simply and directly by means of eqn. (15). For the C—H bond,  $\eta^2$  was found <sup>17</sup> to be insensitive to the value of  $\alpha_{\rm H}^2$ , the total change being only 0.2% over a range of  $\alpha_{\rm H}^2$  from 0.24 to 0.45.  $\eta^2$  is also relatively insensitive to  $\lambda_{C-H}$ . Substituents are expected to change the electronegativity of the C atom by at most 0.1 to 0.2 units according to estimates of effective electronegativities by proton chemical shift measurements.<sup>26</sup> The empirical values of  $\lambda_{B-H}$ ,  $\lambda_{C-H}$  and  $\lambda_{N-H}$  given by Karplus and Grant,<sup>2</sup> indicate that an increase in electronegativity of the carbon by 0.2 units would change  $\lambda_{C-H}$  from 0.374 to about 0.44. This corresponds to a decrease in  $\eta^2$  to about 0.95  $\eta^2$  (CH<sub>4</sub>).

However, the increase in  $\lambda_{\rm C-H}$  is accompanied by an increase of  $Z_{\rm eff}$  for the 2s and 2p electrons of carbon which leads to a decrease in the overlap integrals  $S_s$ ,  $S_p$ , and to an increase in  $\eta^2$ . Thus, the effects tend to cancel, and even though  $\alpha_{\rm H}^2$ ,  $\lambda_{\rm C-H}$ , and the overlap integrals all change with the substituents,  $\eta^2$  is expected to remain about the same for the substituted methanes. This leads to  $J_0 \cong 500$  c/sec and the linear relation in eqn. (16) between  $J_{\rm CH}$  and  $\alpha_{\rm H}^2$ .

### THE ADDITIVITY OF SUBSTITUENT EFFECTS

The additivity relation observed by Malinowski <sup>14</sup> can be derived by means of eqn. (16) providing one assumes that the substituents redistribute the carbon 2s orbital among the four bonds in a particular manner. First of all, the 2s character must be conserved, that is

$$\alpha_{\rm H}^2 + \alpha_{\rm X}^2 + \alpha_{\rm Y}^2 + \alpha_{\rm Z}^2 = 1. \tag{17}$$

Secondly, each atom or group X is assigned a "characteristic affinity for s character",  $\Delta_X$ . Let  $\Delta_X$  be measured with respect to H so that  $\Delta_X$  is positive if the "s affinity" of X is less than H and negative if greater than H. Consider the four bonds to be four equivalent interconnected potential wells of possibly different depths. The difference in the depths of the wells for X and H is defined as  $\Delta_X$ . The 2s orbital will distribute itself among the wells to give a common 2s level, because of their interconnection. Moreover, this common 2s level, and the content of each well, can be obtained very readily via eqn. (17), i.e. by the assumption that the sum of the 2s content of the four wells is unity.

In CH<sub>4</sub> or CX<sub>4</sub> the four wells are all of the same depth so that 2s character is distributed equally among them, and  $\alpha^2 = \frac{1}{4}$ . In CH<sub>3</sub>X, the H wells are deeper than that of X by the amount  $\Delta_X$  which is distributed equally among four bonds so an H well will have  $(\frac{1}{4})\Delta_X$  2s character more than an H well in CH<sub>4</sub>. In general, the H well in CHXYZ will have  $[(\frac{1}{4})\Delta_X + (\frac{1}{4})\Delta_Y + (\frac{1}{4})\Delta_Z]$  2s character more than an H well in CH<sub>4</sub>. Expressed mathematically, this means that for CH<sub>3</sub>X

$$\alpha_{\rm H}^2({\rm CH_3X}) = \alpha_{\rm H}^2({\rm CH_4}) + (\frac{1}{4})\Delta_{\rm X} \text{ or } (\frac{1}{4})\Delta_{\rm X} = \alpha_{\rm H}^2({\rm CH_3X}) - \alpha_{\rm H}^2({\rm CH_4}), \tag{18}$$

and for CHXYZ,

$$\alpha_{\rm H}^2({\rm CHXYZ}) = \alpha_{\rm H}^2({\rm CH_4}) + (\frac{1}{4})(\Delta_{\rm X} + \Delta_{\rm Y} + \Delta_{\rm Z}). \tag{19}$$

By means of eqn. (16),  $\alpha_{\rm H}^2$  can be eliminated from eqn. (18), giving

$$(\frac{1}{4})\Delta_{\rm X}J_0 = J_{\rm CH}({\rm CH_3X}) - J_{\rm CH}({\rm CH_4}) \equiv \delta_{\rm X},$$
 (20)

which in turn converts eqn. (19) into the observed additivity relation, eqn. (6)

$$J_{\text{CH}}(\text{CHXYZ}) = J_{\text{CH}}(\text{CH}_4) + \delta_{\text{X}} + \delta_{\text{Y}} + \delta_{\text{Z}}.$$
 (6)

In addition, a general equation, similar to eqn. (19), may be written for the s character of the carbon orbital in the C—X bond,

$$\alpha_{\mathbf{X}}^{2}(\mathbf{CHXYZ}) = (\frac{1}{4})(1 + \Delta_{\mathbf{X}} + \Delta_{\mathbf{Y}} + \Delta_{\mathbf{Z}}) - \Delta_{\mathbf{X}}.$$
 (21)

#### COMPARISON WITH EXPERIMENT

Experimental values  $^{2, 14}$  of  $J_{\text{CH}}(\text{CH}_3\text{X})$  and the resulting  $\Delta_{\text{X}}$  obtained from them by means of eqn. (20) are given in table 1 for a number of substituents. The  $\Delta_{\text{X}}$  tend to follow the electronegativity of X, being negative for electropositive substituents (-0.096 for Al) and positive for electronegative (+0.2 for the halogens). However, at least another factor is important because for substituents with the same electronegativity,  $\Delta_{\text{X}}$  is larger for those which have the greater number of lone pair electrons.

Moreover,  $\Delta_X$  is virtually the same for the four halogens in spite of their large range of electronegativity. Qualitatively, the  $\Delta_X$  values are consistent with charge and spin correlation effects <sup>17</sup> in CH<sub>3</sub>X, but their detailed significance remains to be determined.

Table 1.—Substituent parameters  $\Delta_{\mathbf{X}}$  obtained from  $J_{\mathrm{CH}}$  observed in some CH3X compounds

$CH_3X$	J <sub>CH</sub> (CH <sub>3</sub> X) c/sec	$^{\alpha^2_{ ext{H}}}$	$\Delta_{\mathbf{X}}$	CH <sub>3</sub> X	J <sub>CH</sub> (CH <sub>3</sub> X) c/sec	$\alpha_{\mathbf{H}}^{2}$	$^{\it \Delta}_{ m X}$
$Al_2(CH_3)_6$	113	0.226	-0.096	CH <sub>3</sub> C≡CH	I 132	0.264	+0.056
$Si(CH_3)_4$	118	0.236	-0.056	$CH_3NH_2$	133	0.266	+0.064
$CH_4$	125	0.250	0.000	CH <sub>3</sub> CCl <sub>3</sub>	134	0.268	+0.072
$CH_3CH_3$	126	0.252	+0.008	CH <sub>3</sub> CN	136	0.272	+0.088
$ ext{CH}_3\phi$	126	0.252	+0.008	CH <sub>3</sub> SH	138	0.276	+0.104
$CH_3CHO$	127	0.254	+0.016	$(CH_3)_2S$	138	0.276	+0.104
$CH_3CH_2Br$	128	0.256	+0.024	ĊH₃ÕH	141	0.282	+0.128
CH <sub>3</sub> CH <sub>2</sub> Cl	128	0.256	+0.024	$CH_3O\phi$	143	0.286	+0.144
$CH_3COOH$	130	0.260	+0.040	$CH_3F$	149	0.298	+0.192
CH <sub>3</sub> CHCl <sub>2</sub>	131	0.262	+0.048	CH <sub>3</sub> Cl	150	0.300	+0.200
$CH_3CH_2I$	132	0.264	+0.056	CH <sub>3</sub> I	151	0.302	+0.208
CH <sub>3</sub> NHCH <sub>3</sub>	132	0.264	+0.056	CH <sub>3</sub> Br	152	0.304	+0.206

The effects of substituents upon  $\alpha_{\rm H}^2$  are additive to within an accuracy of 2% for about 20 polysubstituted methanes. 14, 17 This may be seen in fig. 4 where the observed coupling constants  $J_{\rm CH}$  are plotted against  $\alpha_{\rm H}^2$  values predicted by means of eqn. (19) from the  $\Delta_{\rm X}$  values in table 1. Also plotted in fig. 4 are the  $J_{\rm CH}$  values observed 12, 13, 27 for the 16 unsaturated hydrocarbons listed in table 2. The calculations carried out for the methanes were extended to  $J_{\rm CH}$  in these  $sp^2$  and sp hybridized

Table 2.— $J_{\text{CH}}$  observed in hydrocarbons with  $\mathit{sp}^2$  and  $\mathit{sp}$  hybridization, and values "predicted" for  $\alpha_{\text{H}}^2$  in ethylenes using the  $\Delta_{\text{X}}$  values from substituted methanes

compound	J <sub>CH</sub> c/sec	$\alpha_{\mathbf{H}}^2$	compound	J <sub>CH</sub> c/sec	$\alpha_{\mathbf{H}}^2$
naphthalene	157	$sp^2$	$CH_2 = CCl_2$	166	0.349
benzene	159	$sp^2$	$CH_2 = 13CHCI$	195	0.402
mesitylene	160	$sp^2$	cis CHCl=CHCl	198	0.408
$(CH_3)_2C=C=^{13}CH_2$	166	$sp^2$	trans CHCl=CHCl	199	0.408
cyclohexene	170	$sp^2$	CCl <sub>2</sub> =CHCl	201	0.416
ethylene	157	0.336	$CH_3C=13C-H$	248	sp
$CHCl=13CH_2$ (cis)	160	0.341	$\phi C \equiv 13C - H$	251	SP
CHCl=13CH <sub>2</sub> (trans)	161	0.341	$H$ — $C$ $\equiv C$ — $C$ $\equiv C$ — $H$	259	SD

compounds. Using  $\lambda_{\rm C-H}=0.374$  and the overlap integrals <sup>28</sup> appropriate to the C—H bond distances in ethylene and acetylene, we find  $\eta^2$  for these two compounds to be  $0.987~\eta^2$  (CH<sub>4</sub>) and  $0.977~\eta^2$  (CH<sub>4</sub>) respectively. Moreover,  $\eta^2/\Delta E$  for ethylene and acetylene is affected no more by substituent effects than it is for the methanes. Hence  $J_{\rm CH} \cong 500\alpha_{\rm H}^2$  for  $sp^2$  and sp hybridized carbon, as well as for  $sp^3$ , except for possible effects of the  $\pi$  electrons. There does not appear to be any simple way of estimating the substituent effects upon  $\alpha_{\rm H}^2$  for the cyclic and acetylenic compounds, so the "pure"  $sp^2$  and sp values of  $\frac{1}{3}$  and  $\frac{1}{2}$  are used without correction in fig. 4. The resulting points scatter somewhat more than those for the polysubstituted methanes, but the agreement with the theoretical line is still good.

 $\alpha_{\rm H}^2$  can be estimated for the substituted ethylenes by using the  $\Delta_{\rm X}$  values obtained from the methanes. The main difference is that there are three  $\sigma$  bonds instead of

four. Also, the substituent CYZ in CYZ=CHX has no counterpart in the methanes. However, it seems reasonable to use  $\Delta_{\text{CHYZ}}$  (methane) for  $\Delta_{\text{CYZ}}$  (ethylene). On this basis the s character for a monosubstituted ethylene is given by

$$\alpha_{\rm H}^2({\rm CH_2} = {}^{13}{\rm CHX}) = ({}^{1}_{3})[1 + \Delta_{\rm CH_2} + \Delta_{\rm X}] = \alpha_{\rm H}^2({\rm CH_2} = {\rm CH_2}) + ({}^{1}_{3})\Delta_{\rm X}, \tag{22}$$

which with eqn. (16) gives rise to

$$J_{\text{CH}}(\text{CH}_2 = {}^{13}\text{CHX}) = J_{\text{CH}}(\text{CH}_2 = \text{CH}_2) + (\frac{4}{3})[J_{\text{CH}}(\text{CH}_3\text{X}) - J_{\text{CH}}(\text{CH}_4)].$$
 (23)

Values of  $\alpha_{\rm H}^2$  predicted by means of eqn. (22) are listed for eight substituted ethylenes in table 2 and plotted, as open circles, in fig. 4 against the observed  $J_{\rm CH}$ .

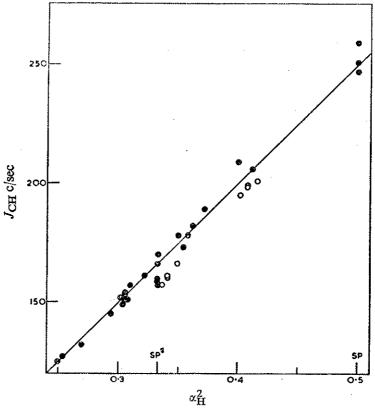


Fig. 4.—Observed  $J_{\rm CH}$  values plotted against predicted values of  $\alpha_{\rm H}^2$ . The straight line is  $J_{\rm CH} = 500~\alpha_{\rm H}^2$  c/sec, upon which all points would fall if the methods for predicting  $\alpha_{\rm H}^2$  were sufficiently accurate. The points for  $sp^2$  and sp hybridization are from table 2, with no confections for substituent effects. The open circles are for the substituted ethylenes in table 2, for which  $\alpha_{\rm H}^2$  was predicted using the  $\Delta_{\rm X}$  values obtained from substituted methanes. The other points represent polysubstituted methanes for which  $\alpha_{\rm H}^2$  was predicted by eqn. (19).

It may be seen that these data are consistently 5 to 10 c/sec below the theoretical line. It seems likely that this discrepancy may result from a  $\pi$  electron contribution to  $J_{\text{CH}}$ . An estimate <sup>17</sup> of  $J_{\text{CH}}^{\pi}$  for ethylene gives a value of -2.6 c/sec, which is of the same sign and magnitude as the discrepancy.

A less satisfactory feature of our results is their relation to observed bond angles. The "interorbital" angles <sup>29</sup> corresponding to the hybridization parameters obtained from  $J_{CH}$  data are consistently smaller than the observed H—C—X and X—C—X angles. In the methyl halides, CH<sub>3</sub>X, the calculated H—C—X angles are about 102° while those observed are 107°; and for CH<sub>2</sub>X<sub>2</sub> the calculated X—C—X angles are 100° and the observed, 112°. In other words the  $\alpha_H^2$  values appear to be too large. These differences, at least in part, could reflect deviations from orbital following <sup>30</sup> of

the same nature as those found in  $CH_2Cl_2$  for which both the H—C—H and the Cl—C—Cl bond angles are greater than tetrahedral. Also, part of the substituent effect,  $\delta_X$ , may result from other than a change in  $\alpha_H^2$ . Interactions between electrons in the C—X bond and those in C—H can contribute to  $J_{CH}$ , without affecting  $\alpha_H^2$ , and have the required additivity. The values for  $\delta_X$  range from -12 to +27 c/sec compared to the 125 c/sec value for  $J_{CH}$  in methane itself. Even relatively small non- $\alpha_H^2$  effects of about 5 c/sec would materially improve the picture. Such contributions might come from the neglected  $O_1$  and  $O_2$  terms <sup>2</sup> and/or from overlap terms <sup>15</sup> which were assumed to be negligible in our calculation of the Fermi contact interaction. Further studies of this question as well as of the nonadditivity of substituent effects found for  $J_{SIH}$  in silanes <sup>16, 17</sup> are indicated.

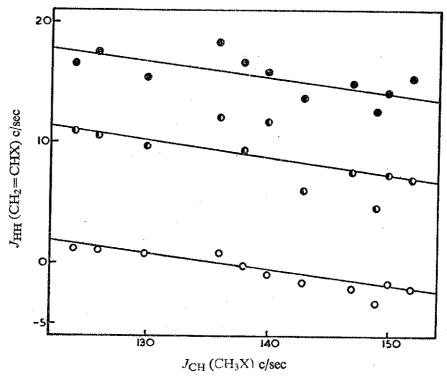


Fig. 5.—The relation between  $J_{\text{CH}}$  observed in CH<sub>3</sub>X and  $J_{\text{HH}}$  observed in CH<sub>2</sub>—CHX. The three sets of points are for  $J_{\text{trans}}^{\text{HH}}$ ,  $J_{\text{cis}}^{\text{HH}}$  and  $J_{\text{gem}}^{\text{HH}}$ , from top to bottom.

# RELATION OF $J_{ m CH}$ TO $J_{ m vic}^{ m HH}$ AND $J_{ m gem}^{ m HH}$

Both  $J_{\rm CH}$  and  $J_{\rm HH}$  in hydrocarbons depend upon the electron density at the proton and on the carbon orbital hybridization so one would expect there to be some relation between the coupling constants. Such a relation is implicit in the fact that  $J_{\rm CH}({\rm CH_3X})^{17}$  and  $J_{\rm HH}({\rm CH_2} = {\rm CHX})$ , 6, 18, 19 cis, trans and geminal, individually have an approximately linear dependence on the electronegativity of X. This may be seen in fig. 5, where the three proton-proton coupling constants observed in a number of substituted ethylenes are plotted against the corresponding  $J_{\rm CH}({\rm CH_3X})$ . The scatter is considerable but there is nonetheless a general linear correlation between  $J_{\rm CH}$  and each of the three types of  $J_{\rm HH}$ . It is noteworthy that the scatter comes mainly from the  $J_{\rm CH}$  values, which indicates that there are interactions affecting  $J_{\rm CH}$  which do not contribute significantly to  $J_{\rm HH}$ .

Another point of interest is that all three types of  $J_{\rm HH}$  increase while  $J_{\rm CH}$  decreases, based upon the arbitrary assignment of  $J_{\rm trans}^{\rm HH}$  as positive. A decrease in  $J_{\rm CH}$  implies a decrease in the s character of the C—H bond. In turn, this would tend to decrease

the C—C—H bond angle. And, according to valence bond calculations of  $J_{\rm HH}$  in the HCCH fragment,<sup>3</sup> this would increase both  $J_{\rm cis}^{\rm HH}$  and  $J_{\rm trans}^{\rm HH}$ , as observed. It is surprising to find virtually the same dependences upon  $J_{\rm CH}$  for all three types of  $J_{\rm HH}$  in spite of the different structural features and magnitudes involved, particularly for  $J_{\rm gem}^{\rm HH}$ . These similar slopes in fig. 5 may be accidental; nonetheless they are one of many features of internuclear coupling which remain to be explained.

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